Gradient Descent: A method used to find the minima of $f : \mathbb{R}^n \to \mathbb{R}$. The algorithm is as follows:

Step 1: Choose an initial point $x^{(0)}$ in $\mathbb{R}^n$.

Step 2: Define $x^{(k)} = x^{(k-1)} - t_k \nabla f(x^{(k-1)})$ for some t_k and for $k = 1, 2, \ldots T$.

Step 3: Choose T "appropriately" or take best possible answer from the above.

1. Let $f(x) = x^2$
   (a) Find $f'(x)$
   (b) Draw a sketch of the curve.
   (c) Let $x_0 = 2$, $t_k = \frac{1}{2^k}$. Compute x_k from gradient algorithm
   (d) Mark x_k on your graph and explain what the gradient descent algorithm is doing.
2. Consider the function $f(x) = x^4 - 4x^2$.
   (a) Calculate f' and f''.
   (b) Using the above draw a rough sketch of the Curve and identify its global minima.
   (c) Write out the steps of the Gradient descent algorithm for an arbitrary x_0.
   (d) For suitable choices of x_0, decide (pictorially) where the gradient algorithm will converge to with the help of sketch that you have drawn.
3. Let $\gamma > 1$ and $f(x_1, x_2) = \frac{1}{2}(x_1^2 + \gamma x_2^2)$.
   (a) Can you guess a minima for f ?
   (b) Draw the level curves of f at levels $1, 10, 100$.
   (c) Let $x^{(0)} = (\gamma, 1)$ and $t_k = \frac{2}{\gamma+1}$ for $k \geq 1$. Calculate $x^{(k)}$ using Gradient Descent algorithm.
   (d) Does $x^{(k)}$ converge and if so where ?
4. Let $f(a, b) = (a + b + 1)^2 + (3a + b - 1)^2$.
   (a) Find the gradient of f.
   (b) Find the critical points of f and check if they are local minima.
   (c) can you identify the global minima of f.
   (d) Let $a^{(0)} = 1, b^{(0)} = 1$ and $t_k = \epsilon$ for $k \geq 1$. Calculate $a^{(k)}$ and $b^{(k)}$ using Gradient Descent algorithm.