1. Verify that the vectors

$$v_1 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \quad , v_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

form an orthogonal basis for $\mathbb{R}^3$ with respect to the dot-product. Also express

$$\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

as a linear combination of v_1, v_2, v_3.

2. Consider $\mathbb{R}^3$ with the dot-product. Let

$$v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

be a nonzero vector in $\mathbb{R}^3$. Find a basis for the subspace $W = v^\perp$.

3. Let $\langle , \rangle$ be an inner product on $\mathbb{R}^n$. For a unit vector $u \in \mathbb{R}^n$ and any vector $v \in \mathbb{R}^n$, we define the *orthogonal projection* of v on u by:

$$\pi_u(v) = \langle v, u \rangle u.$$

Prove that $d(\pi_u(v), v) \leq d(\alpha u, v)$ for any $\alpha \in \mathbb{R}$.

4. (a) Find the orthogonal projection of

$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

onto the line $y = 2x$.
(b) Find the orthogonal projection of

$$\begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \quad \text{onto the line} \quad c \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix},$$

where $c \in \mathbb{R}$.

5. * Can you give a proof of Question 3 using calculus?