1. Let $y = ax^2 + bx + c$. In each of the cases below, draw a rough sketch of y:

   (i) $b^2 - 4ac > 0$ and $a > 0$, (ii) $b^2 - 4ac > 0$ and $a < 0$,

   (iii) $b^2 - 4ac = 0$ and $a < 0$, (iv) $b^2 - 4ac = 0$ and $a > 0$,

   (v) $b^2 - 4ac < 0$ and $a < 0$, (vi) $b^2 - 4ac < 0$ and $a < 0$.
   Distinguish each w.r.t. to y attaining its global maximum or minimum.

2. Let $a \neq 0$, and $z = ax^2 + bxy + cy^2$

   (a) Show that $z = \frac{1}{4a}\left[4a^2(x - \frac{by}{2a})^2 + (4ac - b^2)y^2\right]$.

   (b) Can you identify the critical points of z as (max, min, saddle or ??) when :

      i. $4ac - b^2 < 0$
      ii. $4ac - b^2 > 0$ and $a > 0$
      iii. $4ac - b^2 > 0$ and $a < 0$
      iv. $4ac - b^2 = 0$

   (c) Apply the second derivate test to $z = f(x, y) = ax^2 + bxy + cy^2$ and verify the criteria for critical points obtained above.

3. Let $f(x, y) = x + y + \frac{1}{xy}$ with $x > 0, y > 0$. Decide if the function has a maximum and minimum.

4. Vijayalakshmi, a fruit vendor sells apples and oranges. She wants to order x tons of apples and y tons of oranges, which she gets free from a friend. The minimum order for apples though is 3 tons and the minimum order for oranges id 2 tons. The vendor's wearhouse can hold atmost 10 tons of fruit. She can sell the fruit for

   $$(x - 4)^2 + (y - 4)^2 + y.$$

   How much should she order in order to maximize his profit ?

5. *Extra Credit* Let $f : \mathbb{R} \to \mathbb{R}$. Suppose f is differentiable two times, then show that

   $$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2}f'(\xi),$$

   for any $x, x_0 \in \mathbb{R}$ and ξ is a point between x and x_0. In addition, if the second derivative of f is continuous, $f'(x_0) = 0$, $f''(x_0) < 0$, then show that f has a local maximum at x_0.