1. (*Gradient* $\perp$ *level curve*) Let $f : \mathbb{R}^2 \to \mathbb{R}$, be given by $f(x, y) = x^2 + y^2$
   (a) Draw the level curve corresponding to $f(x, y) = 1$ and the Gradient vector $\nabla f(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Explain your observations.
   (b) Draw the curve $\{g(t) := (\cos(\frac{\pi}{2}t), \sin(\frac{\pi}{2}t)) : t \in [0, 1]\}$.
   (c) Find $h : [0, 1] \to \mathbb{R}$ given by $h(t) = f(g(t))$ and show that $h'(t) = 0$
   (d) Conclude that the Gradient vector $\nabla f(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ is perpendicular to the level curve of f at 1.

2. (*Directional Derivative*) Let $f : \mathbb{R}^2 \to \mathbb{R}$, be given by $f(x, y) = x^2 y$
   (a) Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.
   (b) Find $\nabla f(1, 1)$ and directional derivative $\nabla f(1, 1) \cdot v = \nabla f(1, 1)^T v$, along v, where $v = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.
   (c) Express $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ as directional derivatives along some suitable vector v. Explain the meaning of the term directional derivative.

3. (*Gradient is the direction of steepest ascent*) Let $f : \mathbb{R}^2 \to \mathbb{R}$.
   (a) Let $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ be a unit vector. Express the directional derivative of f along v in terms of $v_1, v_2, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$
   (b) Find the direction v, along which the directional derivative is maximum.

4. Let us meet *Squirmy*, she is a ladybug. One day Squirmy decides to explore the fabulous ICTS campus.
   (a) At first, Squirmy starts walking along the curve C in the $x - y$ plane, given by the equation:
   $$\frac{x^2}{400} + \frac{y^2}{100} = 1.$$
   On reaching $(12, 8)$, she is supposed to start walking in a direction perpendicular to C. What direction should she move in ?
   (b) Next Squirmy jumps into the Kitchen via the windo. She lands on a hot plate where dosai is being made, ***Ouch!!***. She is at a position $(1, 1)$ and suppose the temperature is given by
   $$T(x, y) = 1000\mathrm{e}^{-x^2 - y^4}.$$
   (i) What direction should she move in order to decrease the temperature the fastest ?
   (ii) Instead, she moves in the direction $(2, 1)$. Use a linear approximation to estimate the temperature of the plate when Squirmy gets to $(1.02, 1.01)$.

5. (*Extra Credit*) Let $f : \mathbb{R}^2 \to \mathbb{R}$.
   (a) Suppose f is a differentiable function and $C = f(x_0, y_0)$. Show that the Gradient vector $\nabla f(x_0, y_0)$ is perpendicular to the level curve at C.
   (b) Suppose $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$. Show that $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exists at $(0, 0)$ but f is not continous at $(0, 0)$.