(1) Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$.

(a) Find the eigenvalues of A. Find the corresponding eigenvectors.
(b) Find a basis for the corresponding eigenspaces.

(2) Find eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$. Also, find the corresponding eigenspaces, and bases for these eigenspaces.

(3) Let $v_1, v_2, \ldots, v_k$ be eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$ of a matrix A. Show that $S = \{v_1, v_2, \ldots, v_k\}$ is a linearly independent set.

(4) Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Find a matrix P such that $P^{-1}AP$ is diagonal, and find a formula for A^{30}.

(5) Let P be an invertible $n \times n$ matrix. Show that eigenvalues of a $n \times n$ matrix A are same as the eigenvalues of the matrix $P^{-1}AP$.