

- (1) Calculate the dimension of the column space (or *rank*) of the following matrix.

$$A = \begin{bmatrix} 0 & 16 & 8 & 4 \\ 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \end{bmatrix}$$

- (2) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 0 & 3 \\ -1 & 4 & -3 & 0 \end{bmatrix}.$$

- (a) Compute a basis for the null space of A .
(b) Compute a basis for the range of A .
(c) Compute a basis for the range of A^t .

- (3) Find eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix}$.

- (4) Let A be a $n \times n$ matrix over \mathbb{R} and $\lambda \in \mathbb{R}$ be an eigenvalue of A . Show that the set

$$E_\lambda = \{X \in \mathbb{R}^n : AX = \lambda X\}$$

forms a subspace of \mathbb{R}^n . (This subspace is called the *eigenspace* corresponding to λ).

- (5) * Do A and A^t have the same eigenvalues? The same eigenvectors?