(1) Let

$$v_1 = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}; \quad v_2 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}; \quad v_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}; \quad w = \begin{bmatrix} -6 \\ 13 \\ -2 \end{bmatrix}.$$

Find a linear combination of $v_1$, $v_2$, and $v_3$ that equals $w$.

(2) Determine whether the following three vectors are linearly dependent or linearly independent in $\mathbb{R}^3$.

$$u = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 17 \\ -5 \\ 5 \end{bmatrix}, \quad \text{and} \quad w = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}.$$

If these vectors are linearly dependent, describe a nontrivial linear combination that yields the zero vector.

(3) Determine whether the following vectors form a basis of $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}.$$

(4) Find a basis for the subspace

$$W = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : 3a + 2b - c = 0 \}$$

of the $M_{2\times 2}$, the space of all $2 \times 2$ matrices over $\mathbb{R}$.

(5) * Find a basis for the space of all real $3 \times 3$ symmetric matrices.