1. Let $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$

   (a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial^2 x}$, $\frac{\partial^2 f}{\partial^2 y}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

   (b) Find all critical points of $f(x,y)$ in the plane.

   (c) Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum or a saddle point.
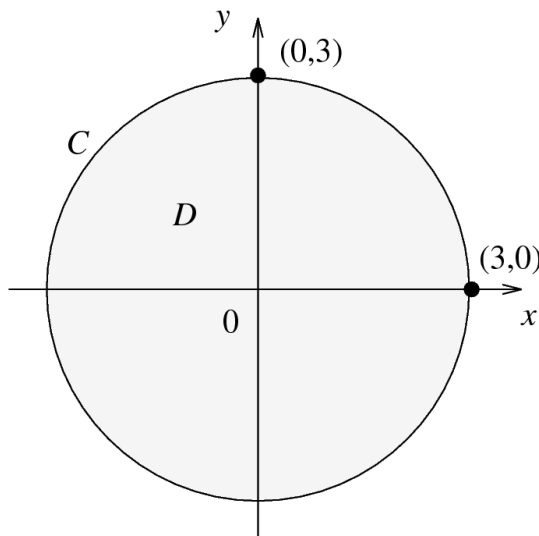
2. Let $f(x,y) = y^2 - 2x^2 y + 2x^3$.

   (a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial^2 x}$, $\frac{\partial^2 f}{\partial^2 y}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

   (b) Find all critical points of $f(x,y)$ in the plane.

   (c) Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum or a saddle point.

   (d) Find the maximum and minium values taken by $f(x,y)$ on the line segment C from $(0,1)$ to $(1,0)$.

3. Let $f(x,y) = x^2 + \frac{2}{3}y^3 + 5y^2 + 8y + 6$

   (a) Are there critical points of f that lie inside C ?

   (b) Are there critical points of f that lie on the circle C ?

4. Vijayalakshmi, a fruit vendor sells apples and oranges. She wants to order x tons of apples and y tons of oranges, which she gets free from a friend. The minimum order for apples though is 3 tons and the minimum order for oranges id 2 tons. The vendor's wearhouse can hold atmost 10 tons of fruit. She can sell the fruit for

$$(x-4)^2 + (y-4)^2 + y$$

How much should she order in order to maximize his profit ?