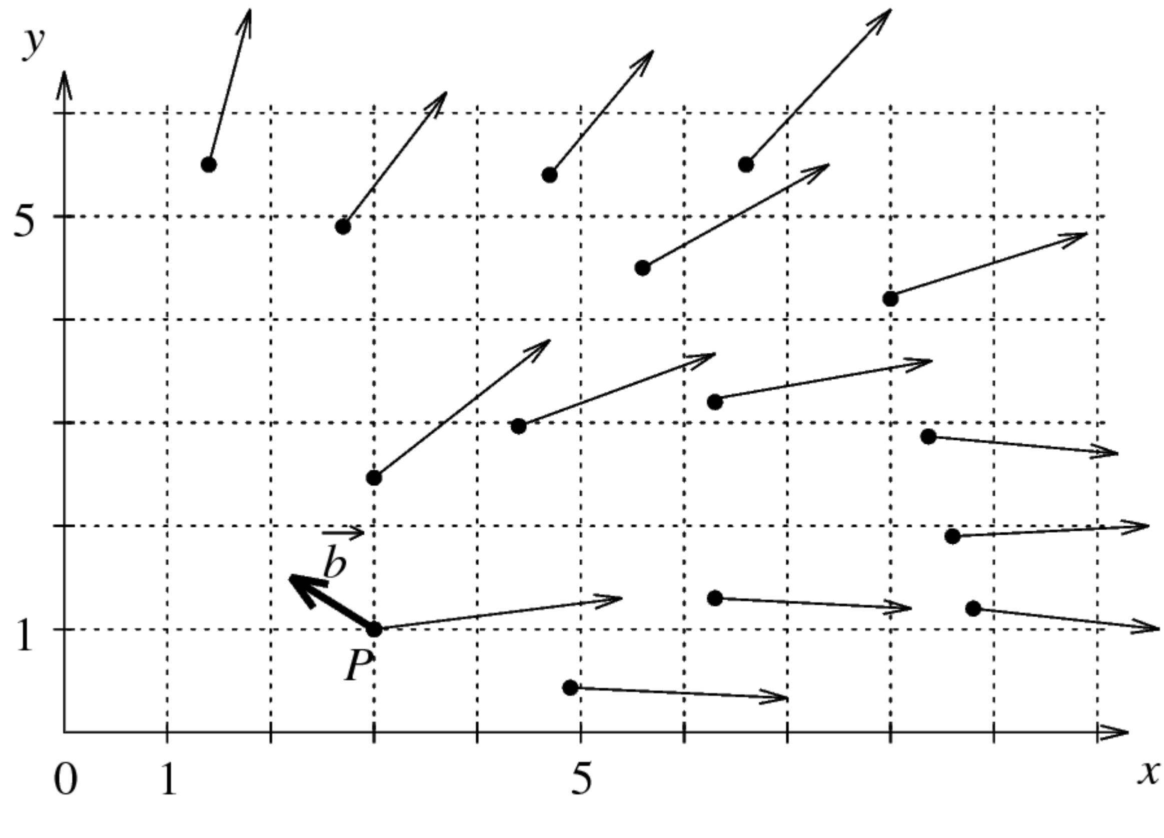
1. Let (x, y) be a point on the $x - y$ plane.
   (a) Write a function $f(x, y)$, giving the square of the distance of (x, y) to the origin.
   (b) Draw the level curves corresponding to $f(x, y) = 0, 1, 4, 9$
   (c) Sketch the intersection of the graph with the $y - z$ plane and with the $x - z$ plane.
   (d) Try and sketch the graph of f.

2. Let $f : \mathbb{R}^n \to \mathbb{R}$

   (a) Suppose $n = 2$ and $f(x, y) = xy + x^2 + y^2$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\nabla f(0, 3)$.
   (b) Suppose $n = 4$, $f(x, y, s, t) = \sin(y + s\sin(t))e^{-(x+s\cos(t))}$. Find $\frac{\partial f}{\partial s}$
   (c) Suppose $n = 2$ and $f(x, y) = \sin(y)e^{-x}$. Find the directional derivative of f along $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
   (d) Suppose $n = 2$, $f(x, y) = e^{x-y}, x(u, v) = \sin(uv)$ and $y(u, v) = u^2 + v^2$. Find the $\frac{\partial f}{\partial u}$ when $(u, v) = (0, 2)$.

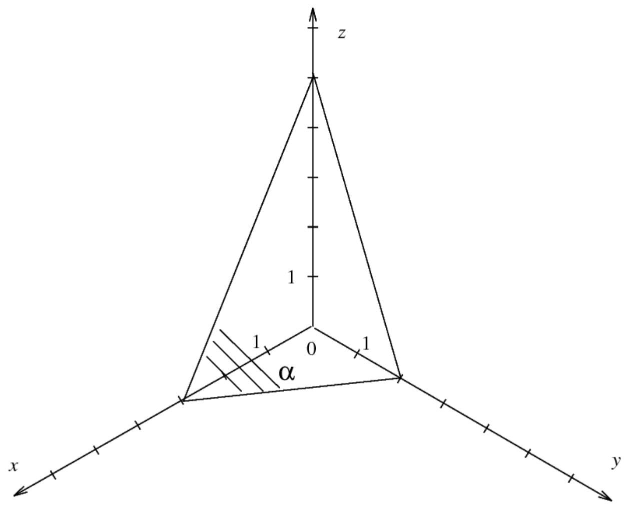
3. $f(x, y)$ is a function of two variables. Below are shown the values of ∇f for some points in the plane. At point P the vector $\overrightarrow{b}$ is in bold is not ∇f.



   (a) Draw a possible level curve for f which goes through at least 4 of the points at which the gradient is shown.
   (b) Does the function increase, decrease or stay the same as you move from $(3, 1)$ to $(5, 0)$?
   (c) Find the directional derivative of f at the point $(3, 1)$ in the direction of $\overrightarrow{b}$.

4. Let m be the line $(1, 2, 1) + t(3, 2, -1)$. Let l be the line $(2, -1, 2) + u(1, 1, 2)$. Find the shortest distance between the two lines.

5. (*Extra Credit*)Let α be the plane $10x + 15y + 6z = 30$ shown below.



Decide with justification whether the following are True, False or Cannot say.

(a) α is normal to the line $(13 + 10t, 15 + 15t, 6 + 6t)$
(b) α is a level surface of the $f(x, y, z) = 10x + 15y + 6z$
(c) α is the graph of the function $f(x, y, z) = 10x + 15y + 6z$
(d) α is the graph of the function $g(x, y) = -\frac{1}{6}(10x + 15y)$
(e) α is the level set of the function $g(x, y) = -\frac{1}{6}(10x + 15y)$

6. (*Extra Credit*) Let $0 \neq v \in \mathbb{R}^2$,$f, g : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} \frac{x_1^3}{x_1^2+x_2^2} & \text{if } (x_1, x_2) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases} \quad \text{and } g(x) = \begin{cases} \frac{x_1 x_2}{x_1^2+x_2^2} & \text{if } (x_1, x_2) \neq (0, 0) \\ 0 & \text{otherwise} \end{cases}$$

(a) Decide if f, g have a directional derivative in the direction v at the origin.
(b) Decide if f, g is continuous at the origin.
(c) Decide whether f, g are differentiable at the origin.