1. Define what is meant by a function $f : \mathbb{R} \to \mathbb{R}$. Determine whether the rules below define functions from $\mathbb{R}$ to $\mathbb{R}$. In each case explain why (or why not) the given rule defines a function from $\mathbb{R}$ to $\mathbb{R}$

   (a) $f(x) =\mid x - 1 \mid$ if $x < 4$ and $f(x) =\mid x \mid -1$ if $x > 2$.

   (b) $f(x) = \frac{((x+3)^2-9)}{x}$ if $x \neq 0$ and $f(x) = 6$ if $x = 0$.

   (c) $f(x) = \sqrt{x^2}$ if $x \geq 2$, $f(x) = 0$ if $0 \leq x \leq 4$,and $f(x) = -x$ if $x < 0$.

2. Define what is meant by domain of a function f. Find the domain of the function:

   (a) $f(x) = 7 + \sqrt{25 - \frac{(x+1)^2}{4}}$.

   (b) $f(x) = \sqrt{x} + \frac{1}{\sqrt{1-x^2}}$.

3. Negate the below statements and express the negations in English,

   (a) Every student in this class has taken Mathematics or Physics in Class XII.

   (b) Every student in this class has taken Mathematics and Biology in Class XII.

   (c) All classrooms in the main building have at least one chair that is broken

   (d) No classroom in the ground floor has only chairs that are not broken.

   (e) In every college there is a student who has taken neither Mathematics nor Biology in high school.

4. Consider the following statements:

   (a) $f(x, y) \neq 0$ whenever $x \neq 0$ and $y \neq 0$.

   (b) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that $\mid f(x) \mid \geq M$.

   (c) For all $M \in \mathbb{R}$ there exists $x \in \mathbb{R}$ such that for all $y > x$ we have $f(y) > M$.

   (d) For all $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(y) > f(x)$.

   (e) For every $k \geq 1$ there exists $x_0 \in \mathbb{R}$ such that $\mid f(x) \mid < \frac{1}{k}$ for all $x > x_0$.

   (f) For every $\epsilon > 0$ there exists $\delta > 0$ such that $\mid f(x) - f(y) \mid < \epsilon$ whenever $\mid x - y \mid < \delta$.

   (i) For each statement (a) - (g) above, give an example of function that satisfies the conditions given in the statement. (ii) Provide the negation of each of the above statements.

5. (*Extra Credit*) Let A and B be finite sets. Define what is meant by 1-1 $f : A \to B$ and what is meant by an onto function from $f : A \to B$.

   (a) Find the total number of functions from A to B.

   (b) Does there always exists a $1 - 1$ and onto function from A to B ?

   (c) Suppose $\mid A \mid=\mid B \mid$ find the number of onto functions from A to B.