(1) Determine all proper subspaces of $\mathbb{R}^2$. Show that distinct proper subspaces have only the zero vector in common.

(2) Find a basis for the space of all real $n \times n$ symmetric matrices.

(3) Let $W \subseteq \mathbb{R}^4$ be the space of solutions of the system of linear equations $AX = 0$, where

$$A = \begin{bmatrix} 2 & 1 & 2 & 3 \\ 1 & 1 & 3 & 0 \end{bmatrix}.$$

Find a basis for W.

(4) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 1 & -1 & 1 & 1 \\ 2 & 1 & 0 & 3 \\ -1 & 4 & -3 & 0 \end{bmatrix}.$$

(a) Compute a basis for the range of A.
(b) Compute a basis for the range of A^t.
(c) Compute a basis for the null space of A.

(5) Calculate the dimension of the column space (or *rank*) of the following matrix.

$$A = \begin{bmatrix} 0 & 16 & 8 & 4 \\ 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \end{bmatrix}$$

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