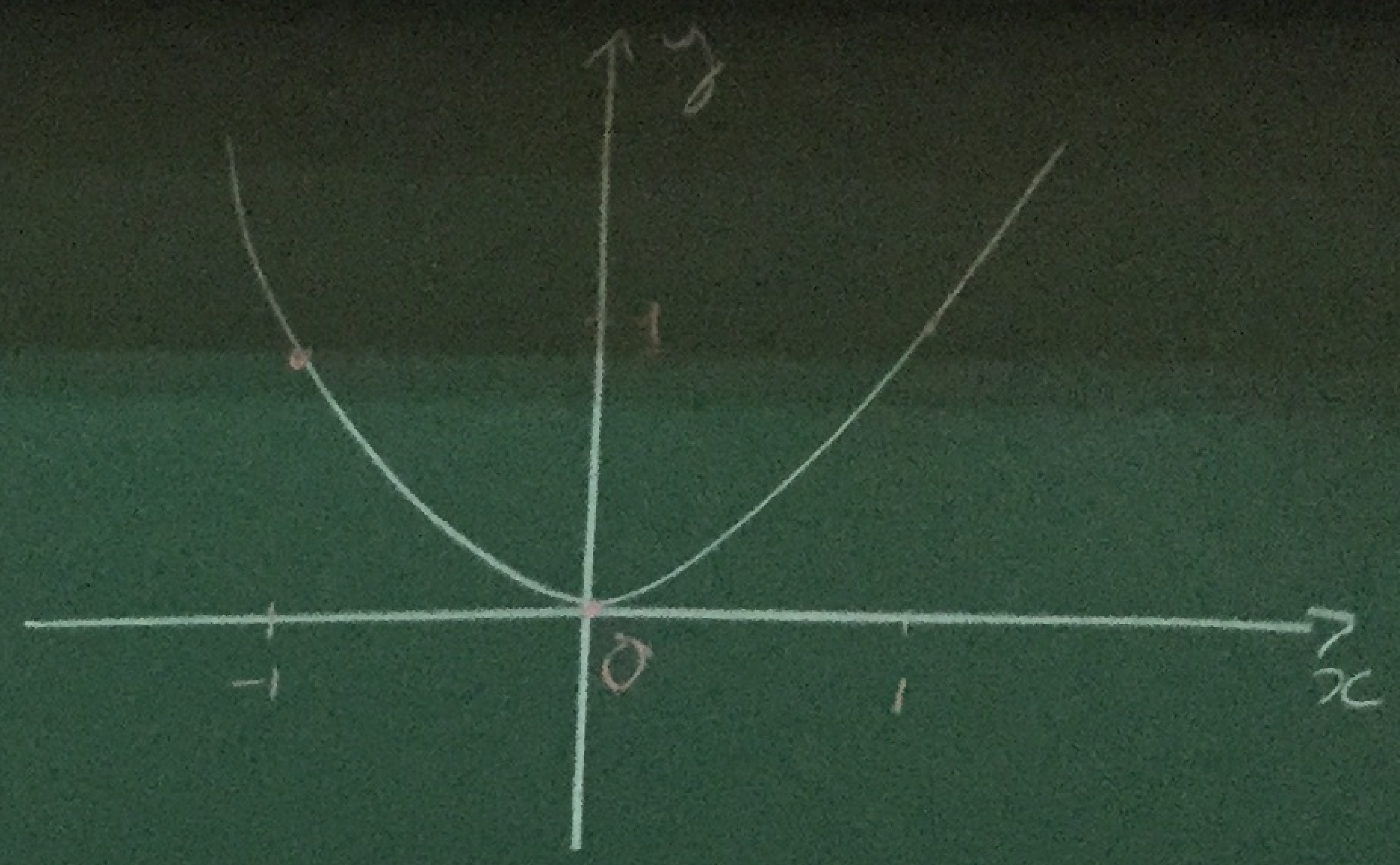
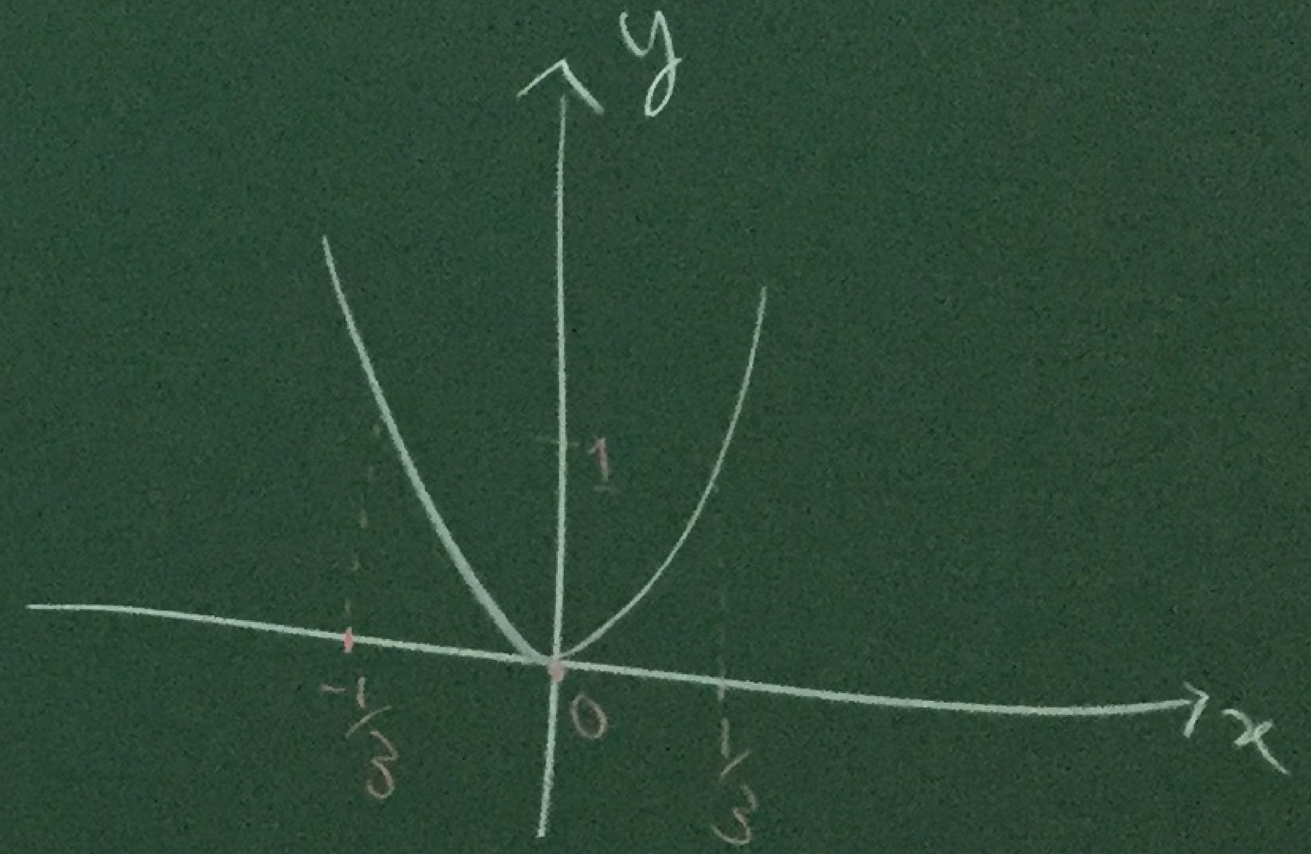
DO NOT CROSS

1 $y = x^2$



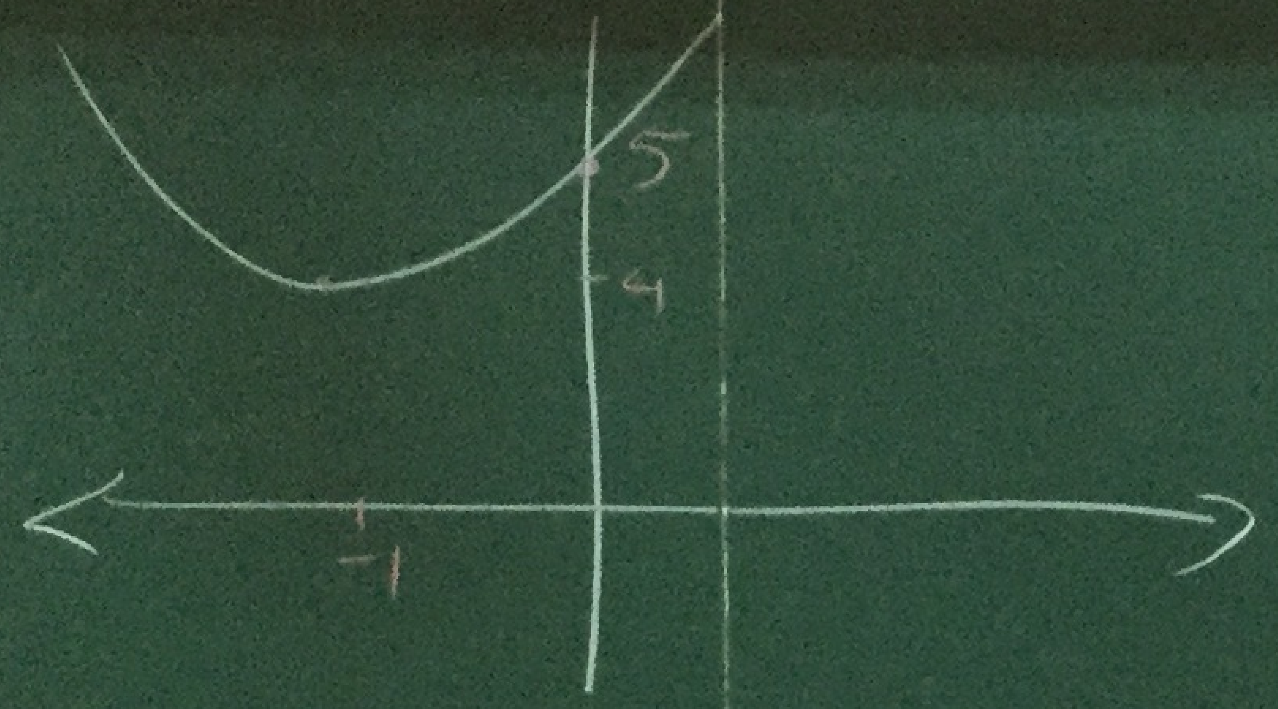
(b)

(a) $y = (3x)^2$

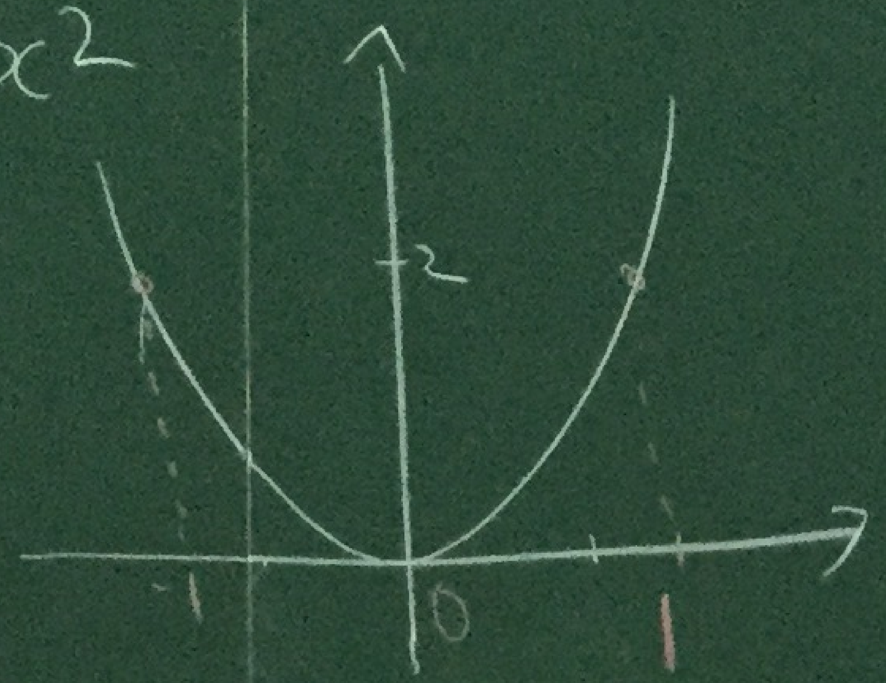


(c)

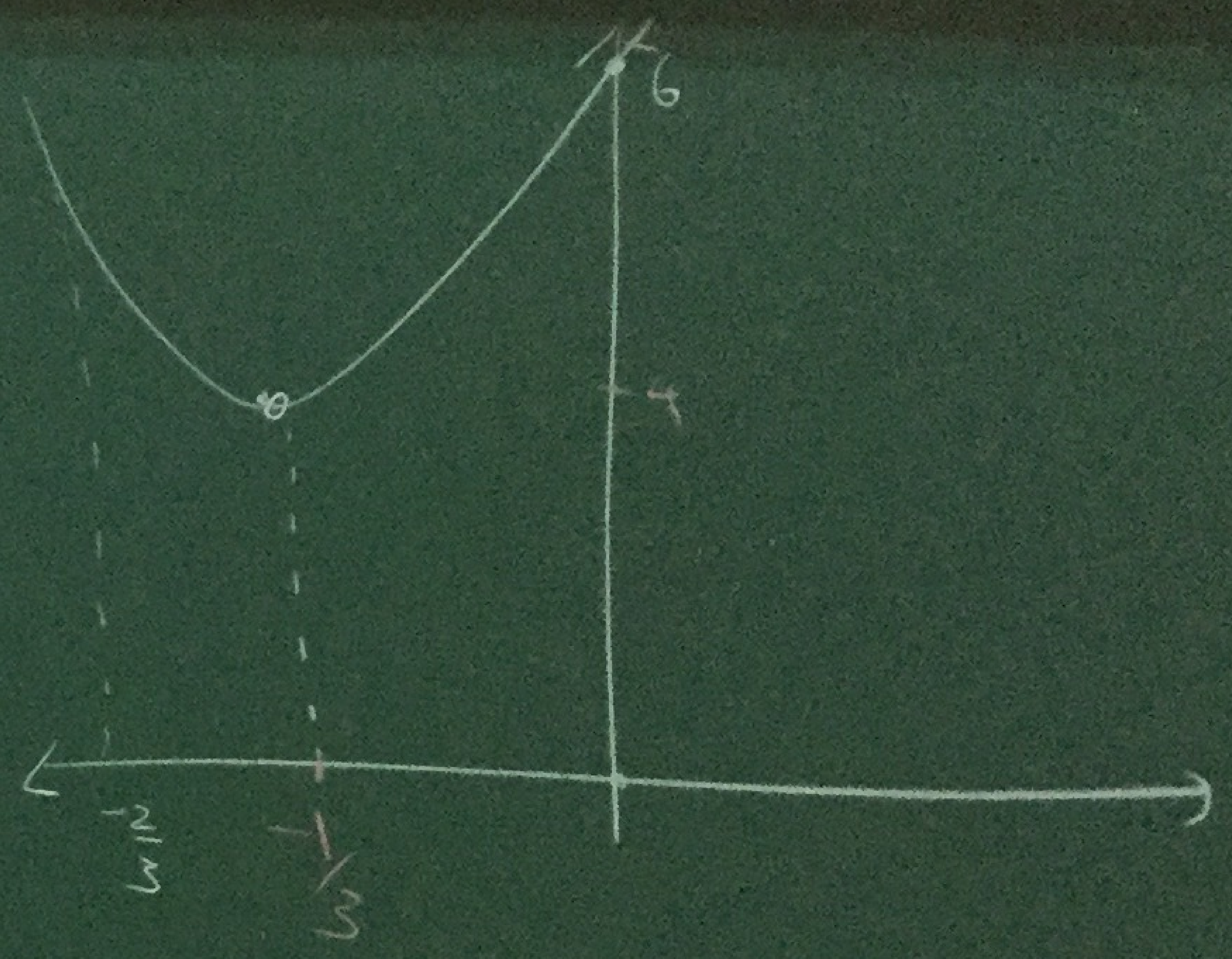
(b) $y = (x+1)^2 + 4$



(c) $y = 2x^2$



(d) $y = 2f(3x+1) + 4$



$y = f(x)$

$y = f(Ax)$ – "Shrink" / scale by $\frac{1}{A}$

$y = f(x+B) + C$ – Shift x to left by B

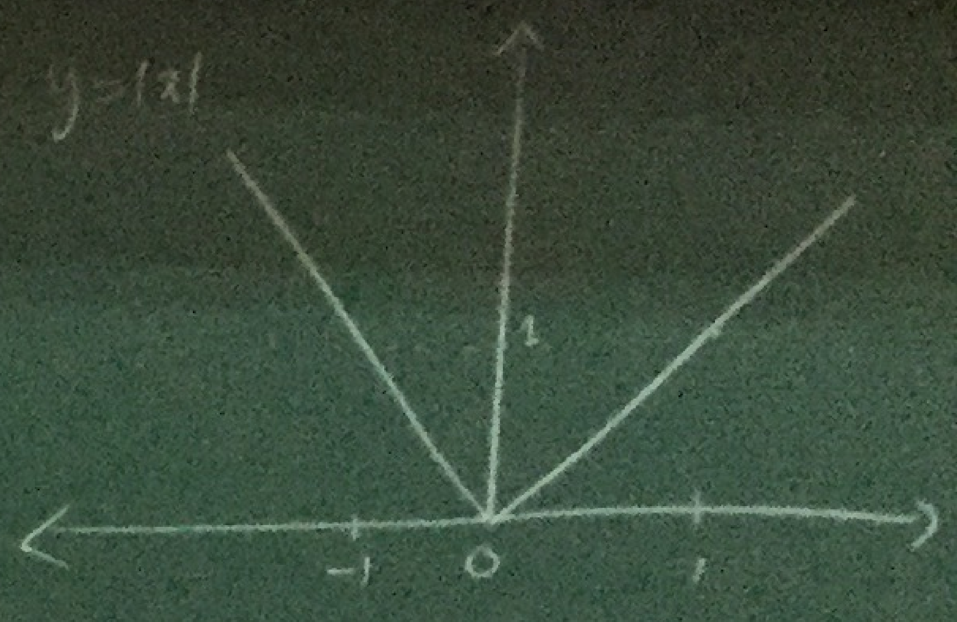
– lift y up by C

$y = Df(x)$ – graph shrink by D in y-axis

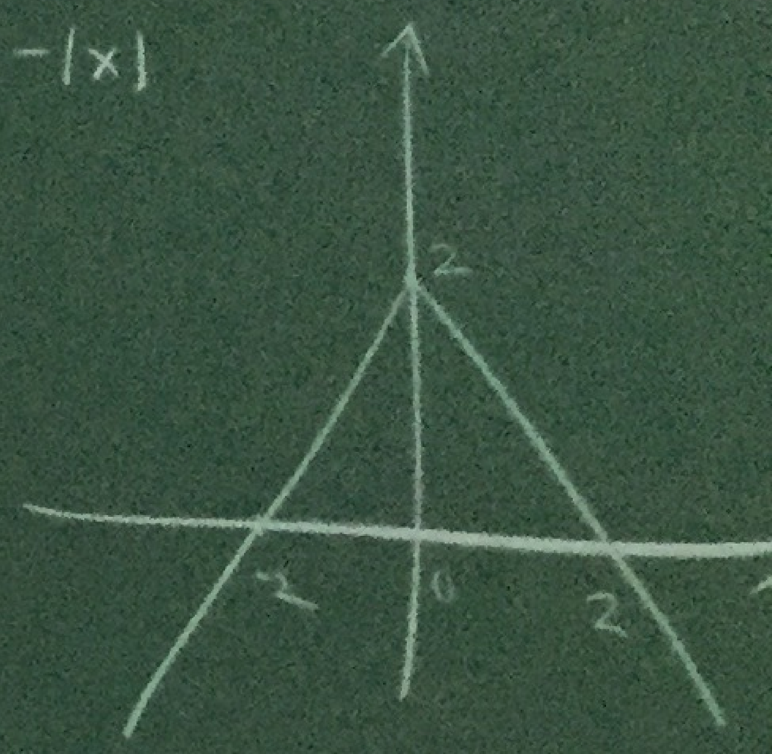
$$\boxed{y = Af(Bx+C)+D}$$

②

$y = |x|$

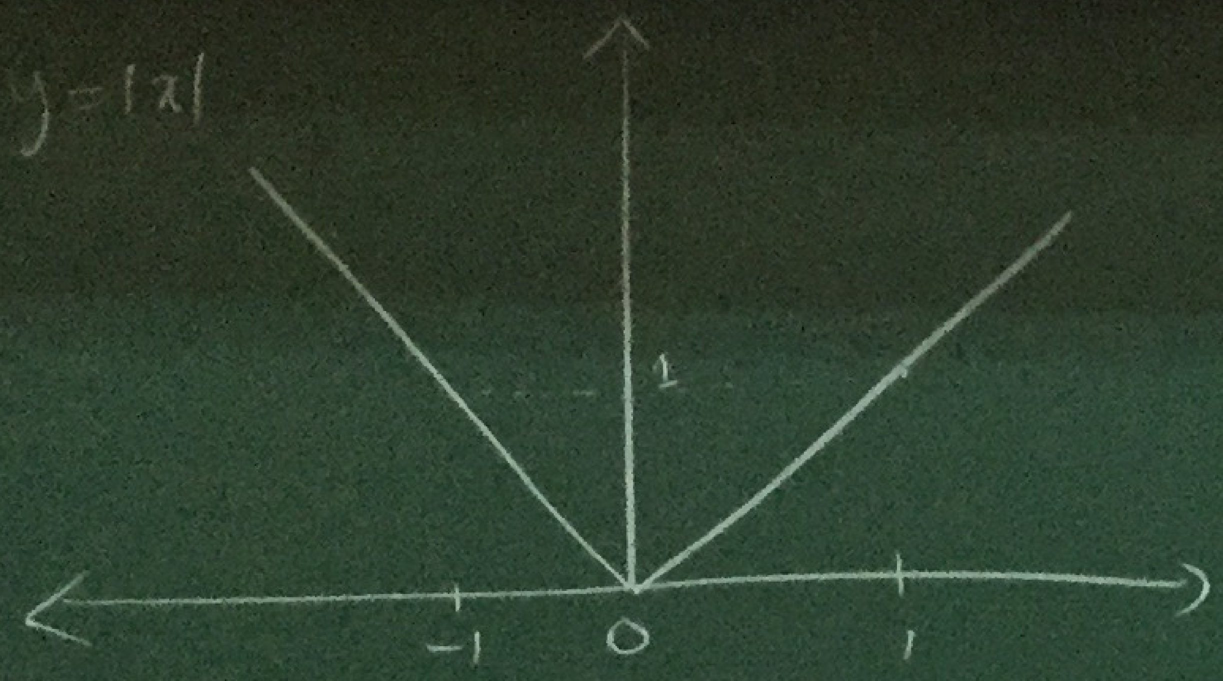


$y = 2 - |x|$



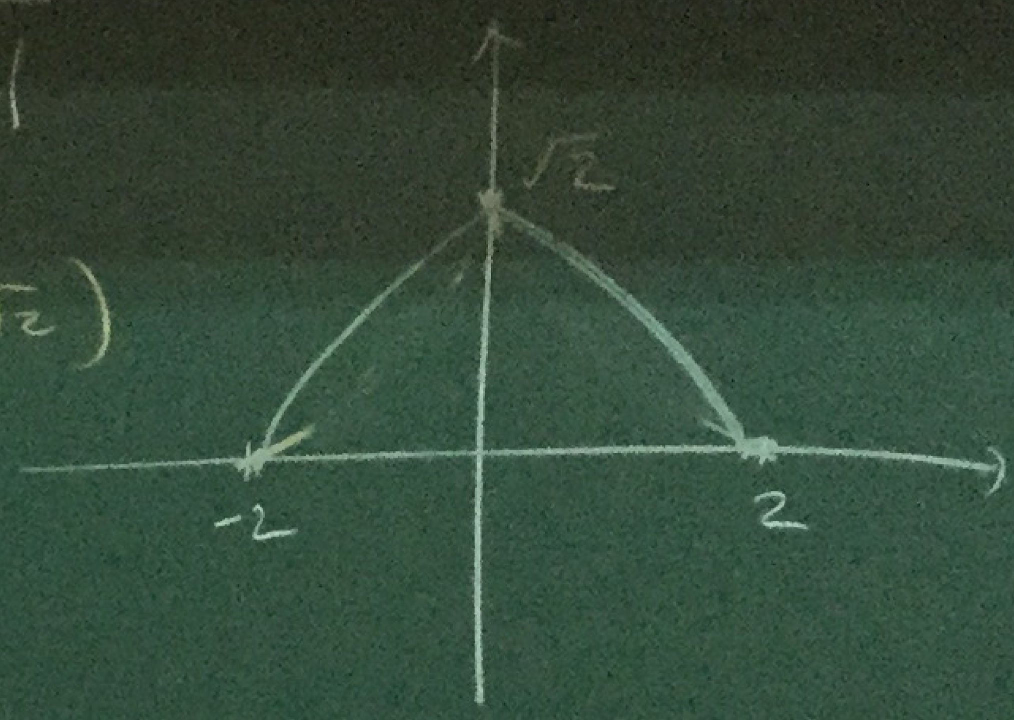
2

$y=|x|$

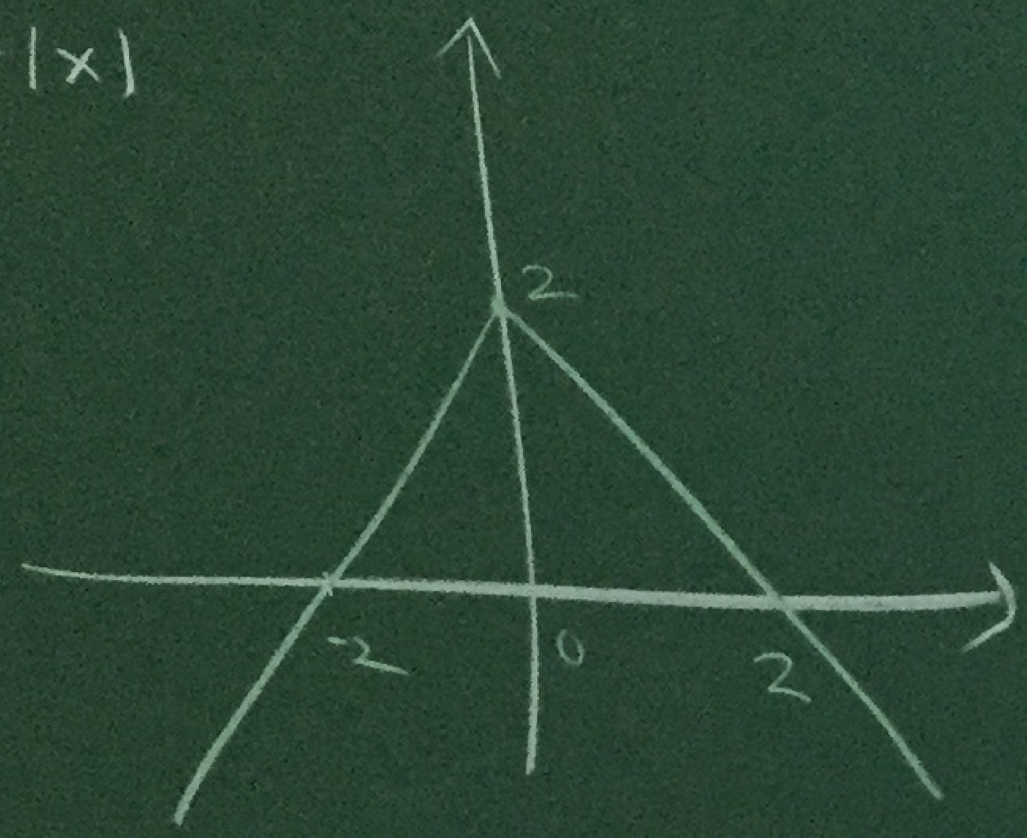


$y = \sqrt{2-|x|}$

"corner" $= (0, \sqrt{2})$



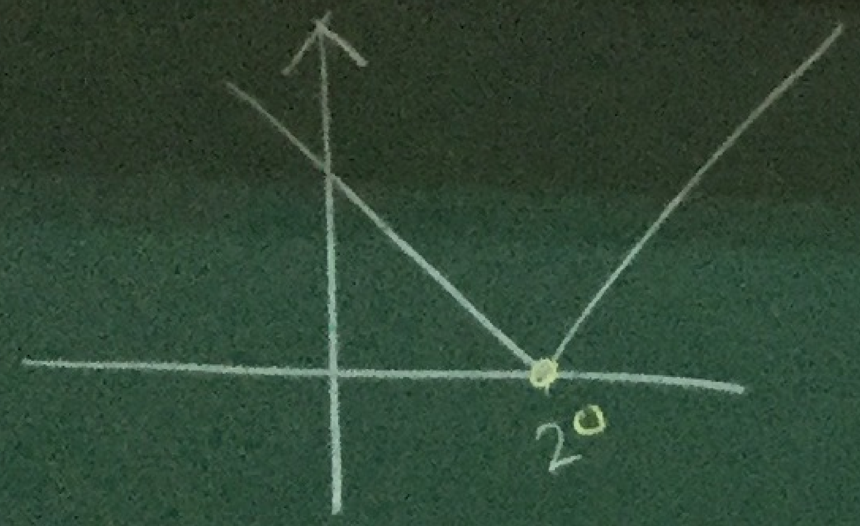
$y = 2-|x|$



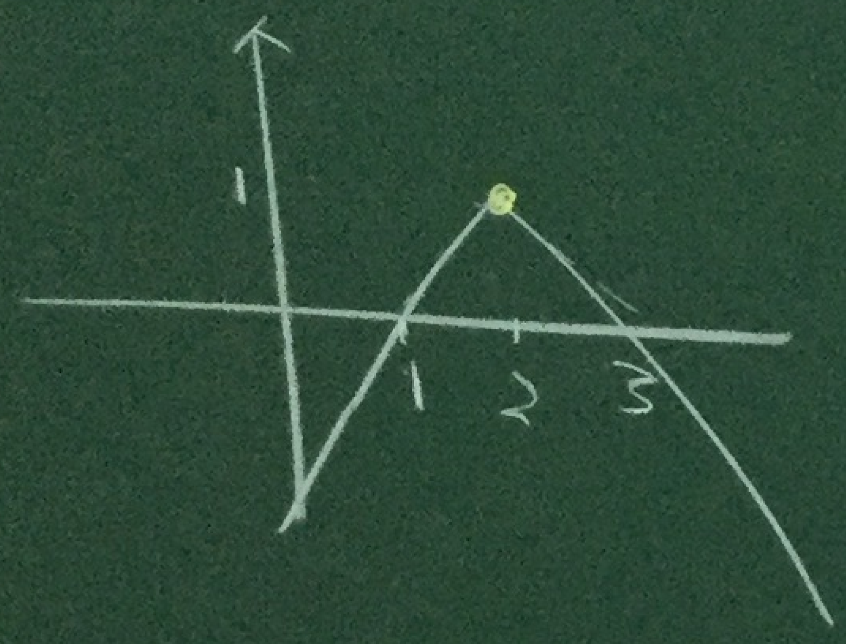
$$y = 6+3\sqrt{1-|x-2|} = 6+3\sqrt{(2-|x-2|)-1}$$

Ans. $(2,9)$ is the new place for corner.

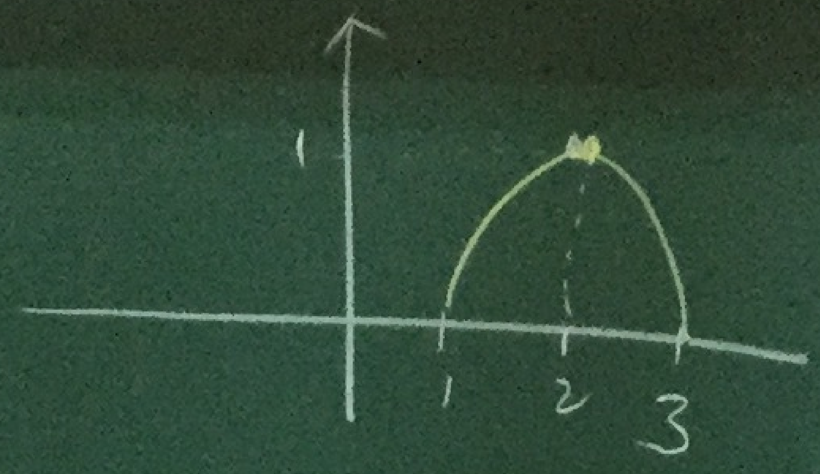
$y = |x-2|$



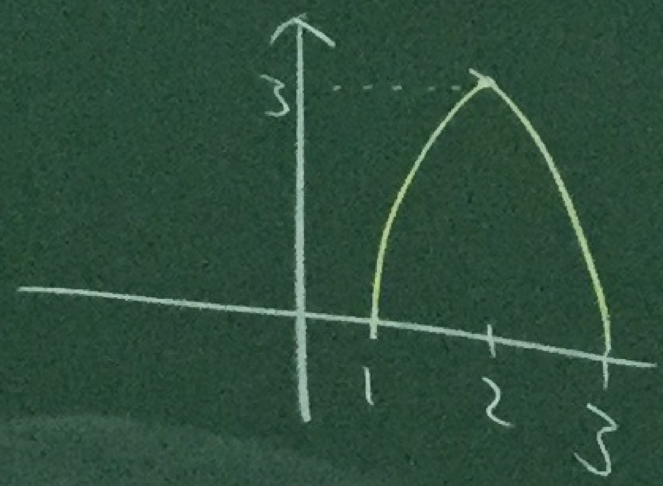
$y = 1-|x-2|$



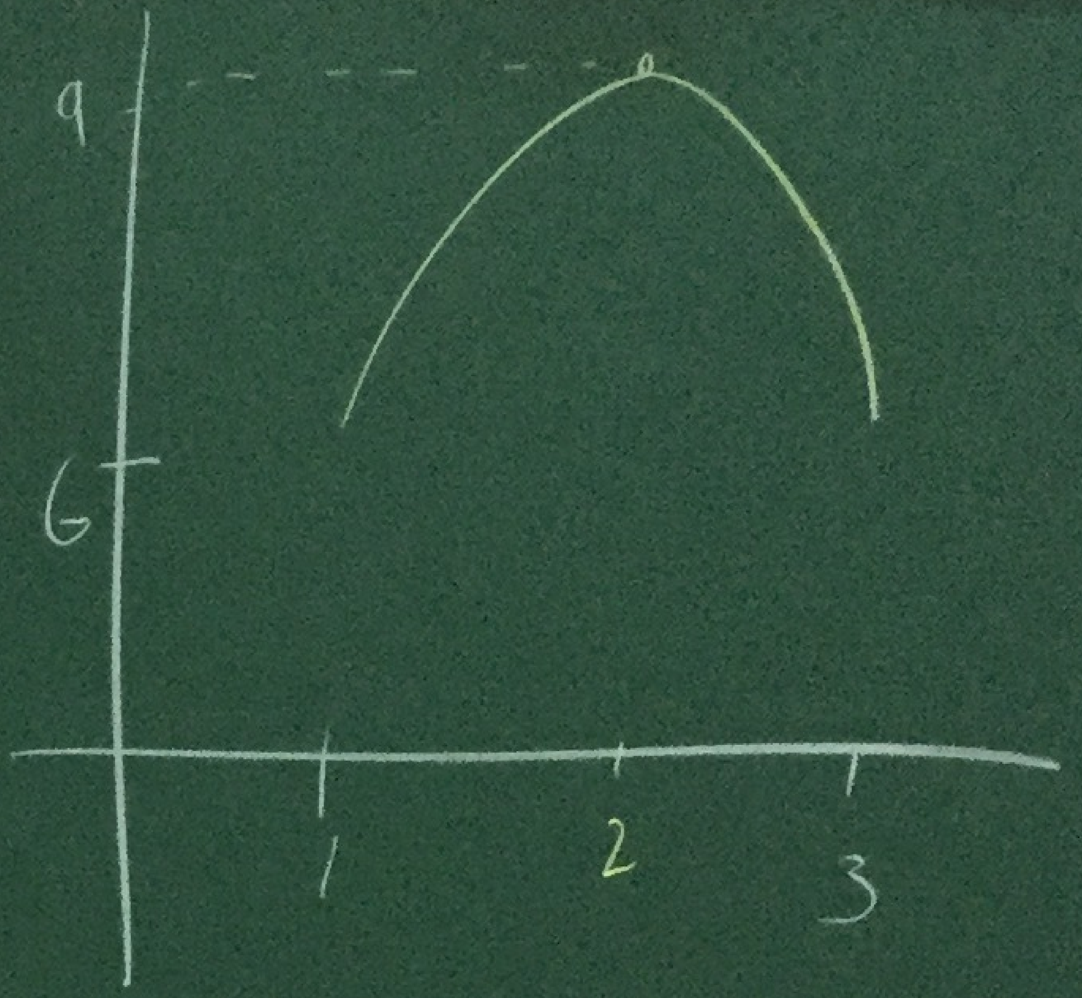
$y = \sqrt{1-|x-2|}$



$y = 3\sqrt{1-|x-2|}$



$y = 6+3\sqrt{1-|x-2|}$

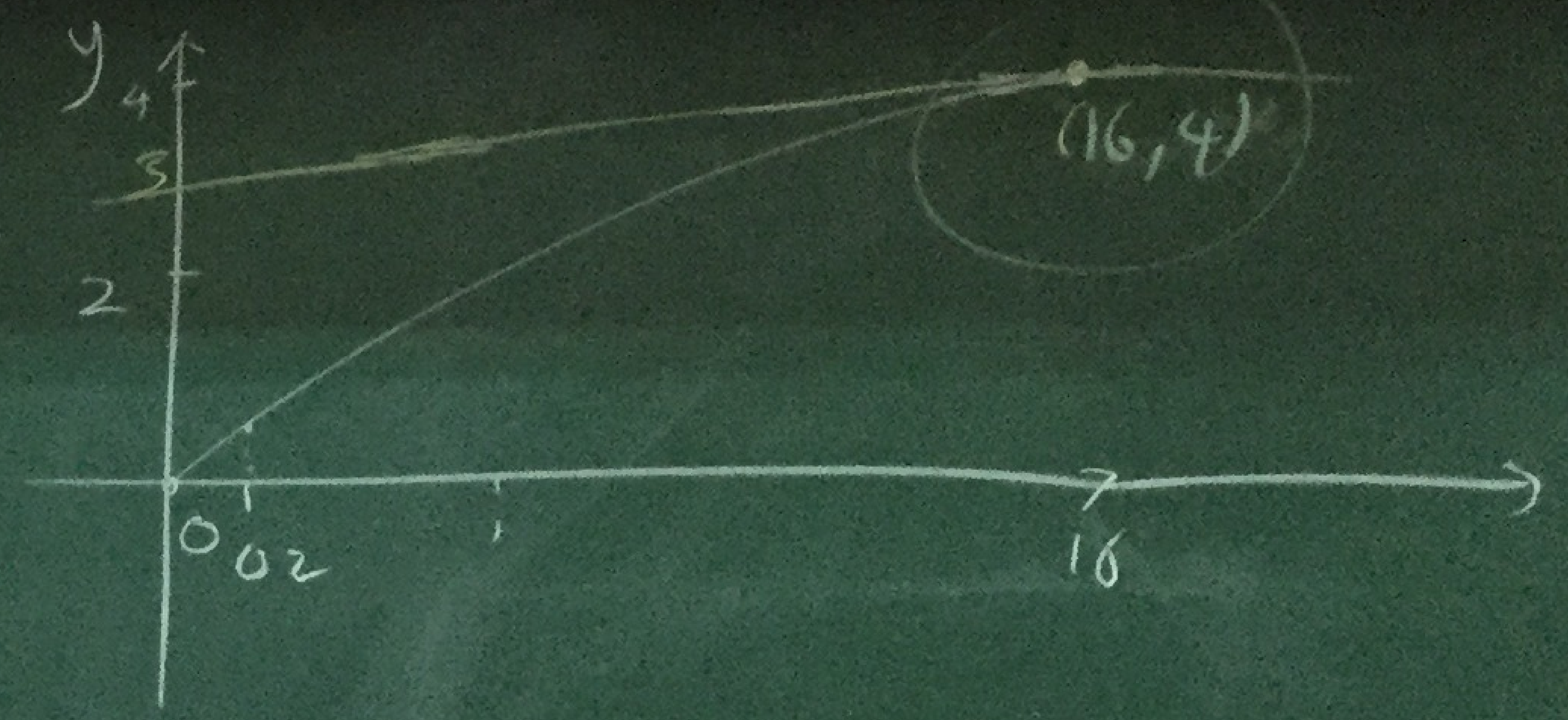


corner is at $(2,9)$

-1

DO NOT CROSS

③ $y = 2x^{1/4}$



$\boxed{\text{Tangent line}}$

$m = f'(16)$

$f'(x) = 2\left(\frac{1}{4}\right)x^{\frac{1}{4}-1}$

$= 2(1/4)x^{-3/4}$

$f'(16) = 2(1/4)(16)^{-3/4} = 1/16$

$\Big\}\ m = 1/16$

$y - 4 = m(x - 16)$

$y = 4 + \frac{1}{16}(x - 16)$

Estimate $f(16.2)$

- Tangent line at $(16,4)$ is a "good" approximation to f "near" $x=16$

$$\boxed{\begin{array}{l} y = mx + c \\ y = f(x) \end{array}}$$

$$y = 4 + \tfrac{1}{16}(x-16)$$

At $x = 16.2$
value of tangent line is

$$y = 4 + \tfrac{1}{16}(16.2-16) = 4 + \tfrac{0.2}{16} = 4 + \tfrac{1}{80}$$

$$\boxed{\begin{array}{l} f(x) = f(a) + (x-a)f'(a) \\ \qquad + R_2(a) \\ \text{Aside: Taylor's Expansion} \end{array}}$$

4

We will take

$$f(16.2) \simeq 4 + \tfrac{1}{80}$$

$f(0.2)$ cannot be estimated by tangent line at $x=16$ because

- at $x = 0.2$, the tangent line is close to 3
- clear that $x = 0.2$ the function is less than 1

to 3
s, than 1

④

I f'

II f''

III f

I f

II f''

III f'

$y = 6+$

s. (2,9)

① $x^2 + xy + y^2 = 7$

Implicit $f(x,y) = 0$

Explicit $y = f(x)$

(Justify ?) $\dfrac{\text{differentiate}}{\text{w.r.t } x}$

$$2x + \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

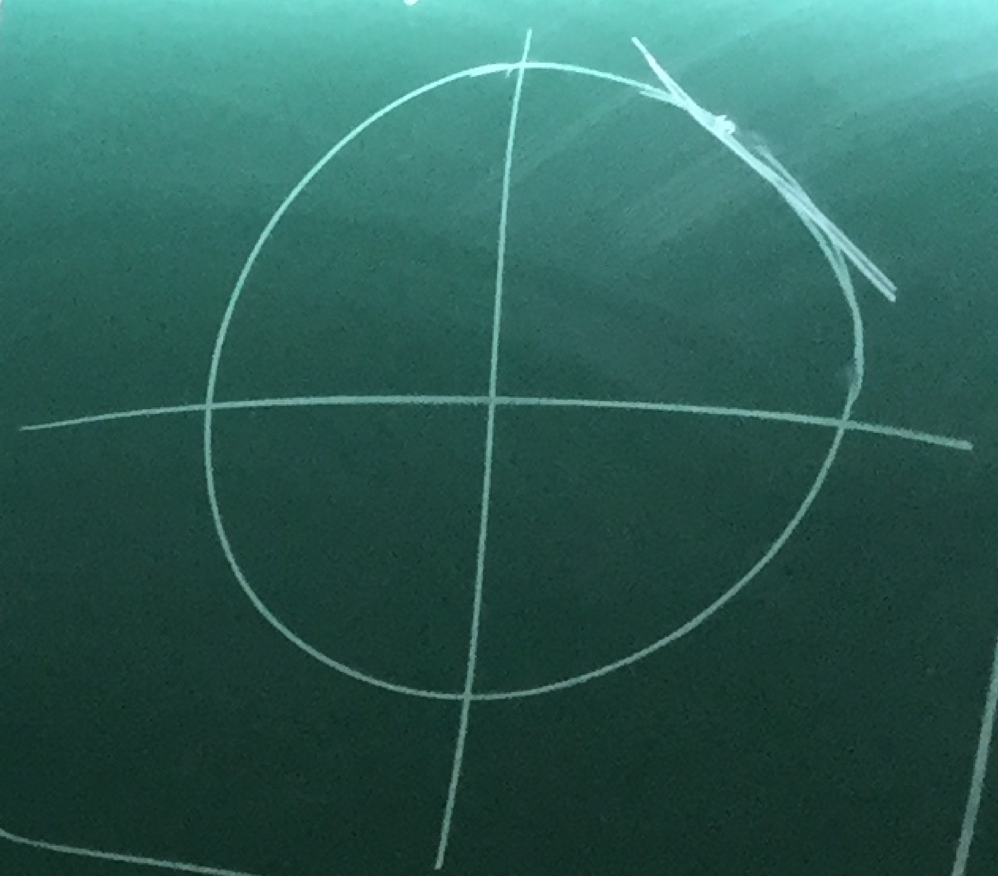
At $(2,1)$

$$2(2) + 1 + 2\frac{dy}{dx} + 2(1)\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{5}{4}}$$

$x + y = 4$

Tangent at $(2,1)$ to curve $x^2+xy+y^2=7$



$m = -5/4$; $(y-1) = -5/4\,(x-2)$

At (

$y=$

"goo

x

$\therefore$ $y=$

(1) $x^2 + xy + y^2 = 7$

Implicit	$F(x,y)=0$
Explicit	$y = f(x)$

$x^2+y^2=4$

(Justify ?) $\dfrac{\text{differentiate.}}{\text{w r t } x}$

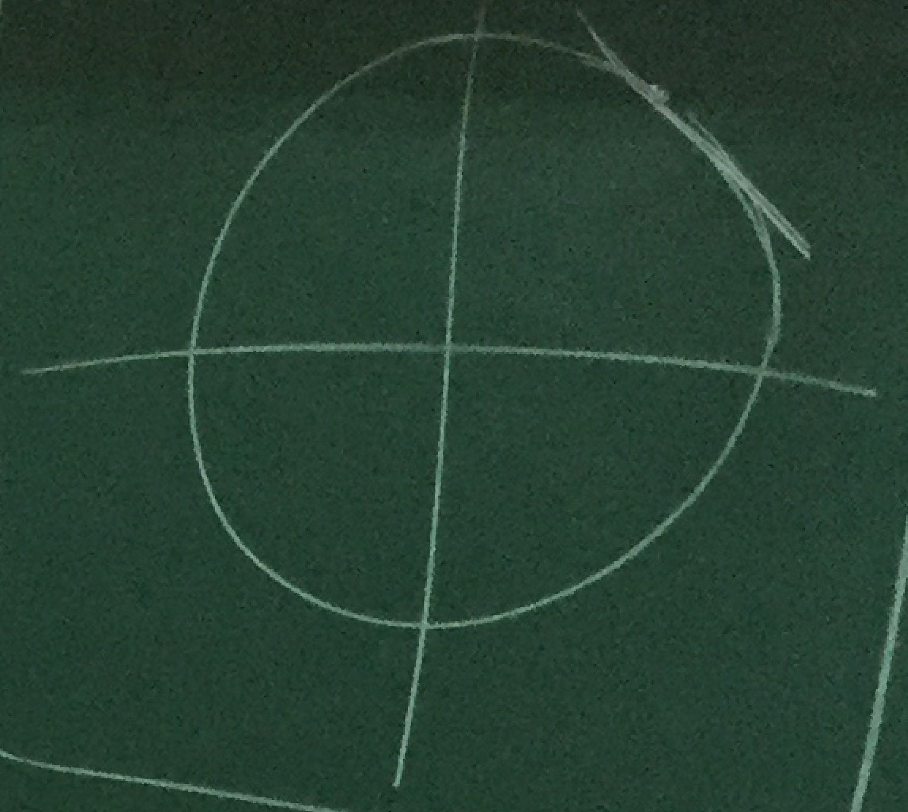
$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

At (2,1)

$$2(2) + 1 + 2\frac{dy}{dx} + 2(1)\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{5}{4}}$$

Tangent at (2,1) to curve $x^2+xy+y^2=7$



$m = -5/4$; $(y-1) = -5/4\,(x-2)$

At (2,1)

$y = 1$

"good"

$x^2 +$

$\therefore\ y = 1$

$=$

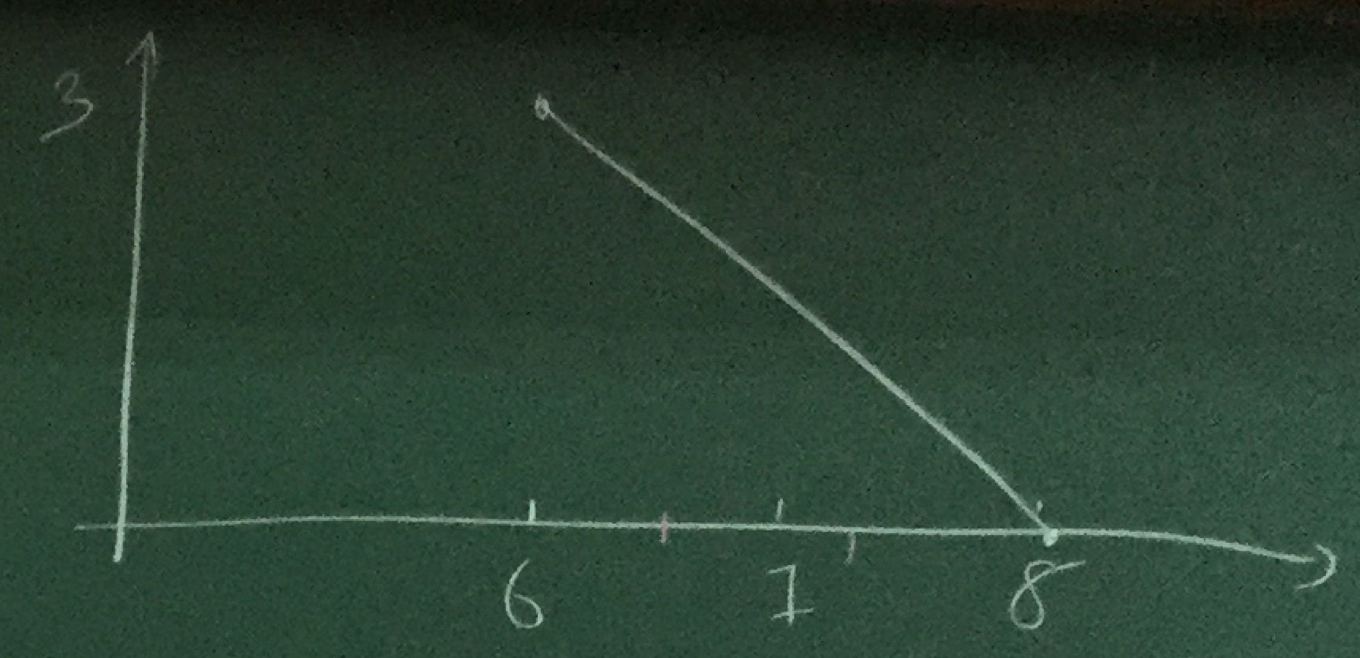
At $(2,1)$

$y = 1 - \frac{5}{4}(x-2)$ is a "good" approximation to $x^2 + xy + y^2 = 7$ "near" $(2,1)$

$\therefore\ y = 1 - \frac{5}{4}(2.01-2)$

$= 1 - \frac{1}{80} = \frac{79}{80}$ is a "good" approximation for value at $x = 2.01$ of the curve $x^2 + xy + y^2 = 7$

2 (a)



- Need to know values of f "near" 7
- Graph is a straight line from $(6,3)$ to $(8,0)$.

$\Rightarrow f(x) - 0 = \frac{0-3}{8-6}(x-8)$

$f(x) = -\frac{3}{2}x + 12$

$\Rightarrow \frac{f(x)}{x} = -\frac{3}{2} + \frac{12}{x}$

$$\text{"}\lim_{x\to 7} \frac{f(x)}{x}\text{"} = -\frac{3}{2} + \frac{12}{7}$$

$$= 3/14$$

$x \in (6, 8)$

$x \in (6, 8)$

$\lim_{x\to a} g(x) = g(a)$ if g is continuous at $x=a$

$g(x) = \begin{cases} x & x \neq 3 \\ 25 & x = 3 \end{cases}$

$\lim_{x\to 3} g(x) = 3$

(b) f is not continuous at $x = 2$

as $f(2) = 0$

$\lim_{x\to 2, x<2} f(x) = 1$

$\lim_{x\to 2, x>2} f(x) = 1$

$(-2, 2) \cup (2, 12)$

(c)

$- f$

f is not c…
at x…

(c) $D = (-2, 2) \cup (2, 6) \cup (6, 8) \cup (8, 10) \cup (10, 12)$

- f is not differentiable at

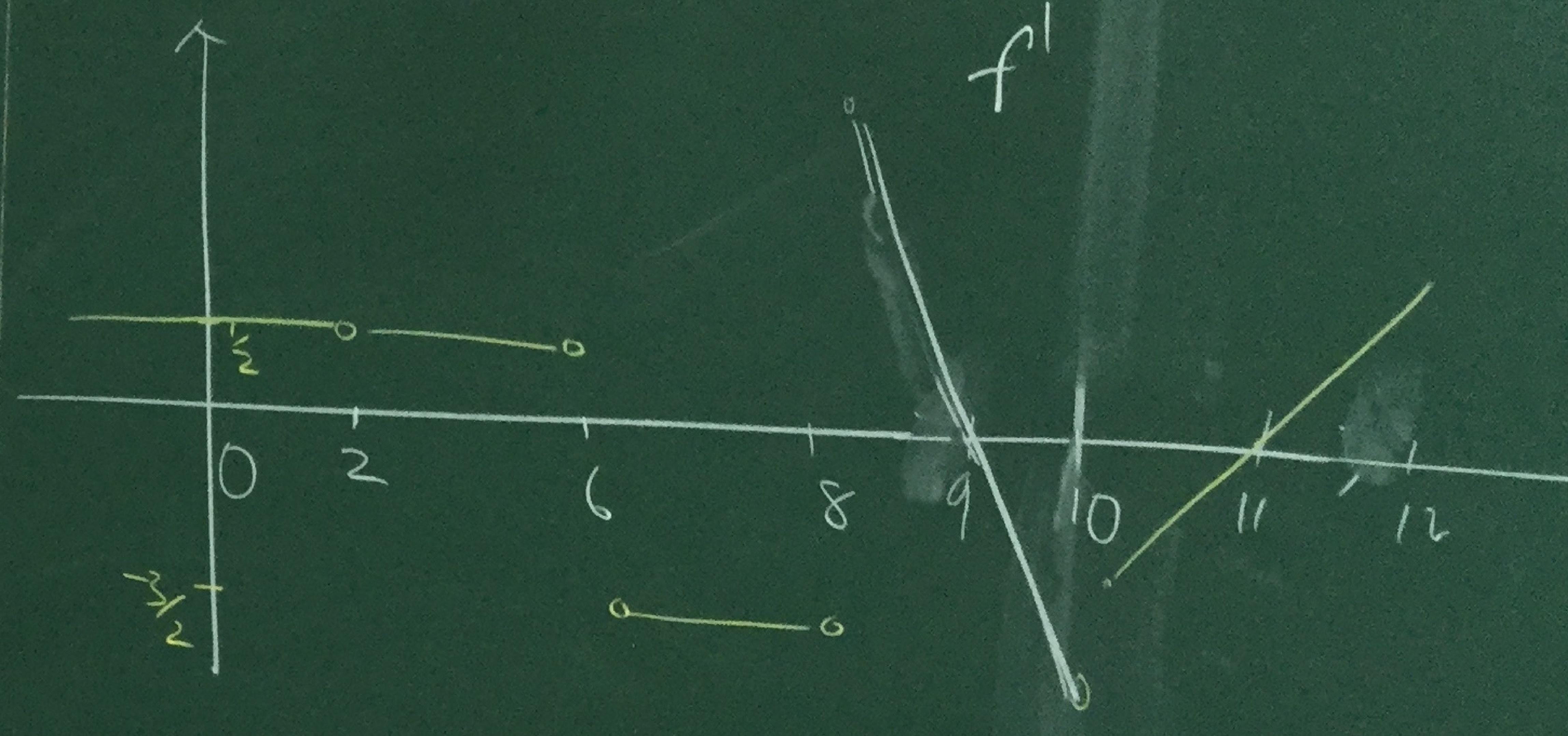
$x = 2$, $x = 6$, $x = 8$, $x = 10$,

f is not continuous at $x = 2$

LHD of $f = \frac{1}{2}$ at $x = 6$

RHD of $f = -\frac{3}{2}$ at $x = 6$

from picture



(c) $D = (-2,2) \cup (2,6) \cup (6,8) \cup (8,10) \cup (10,12)$

- f is not differentiable at

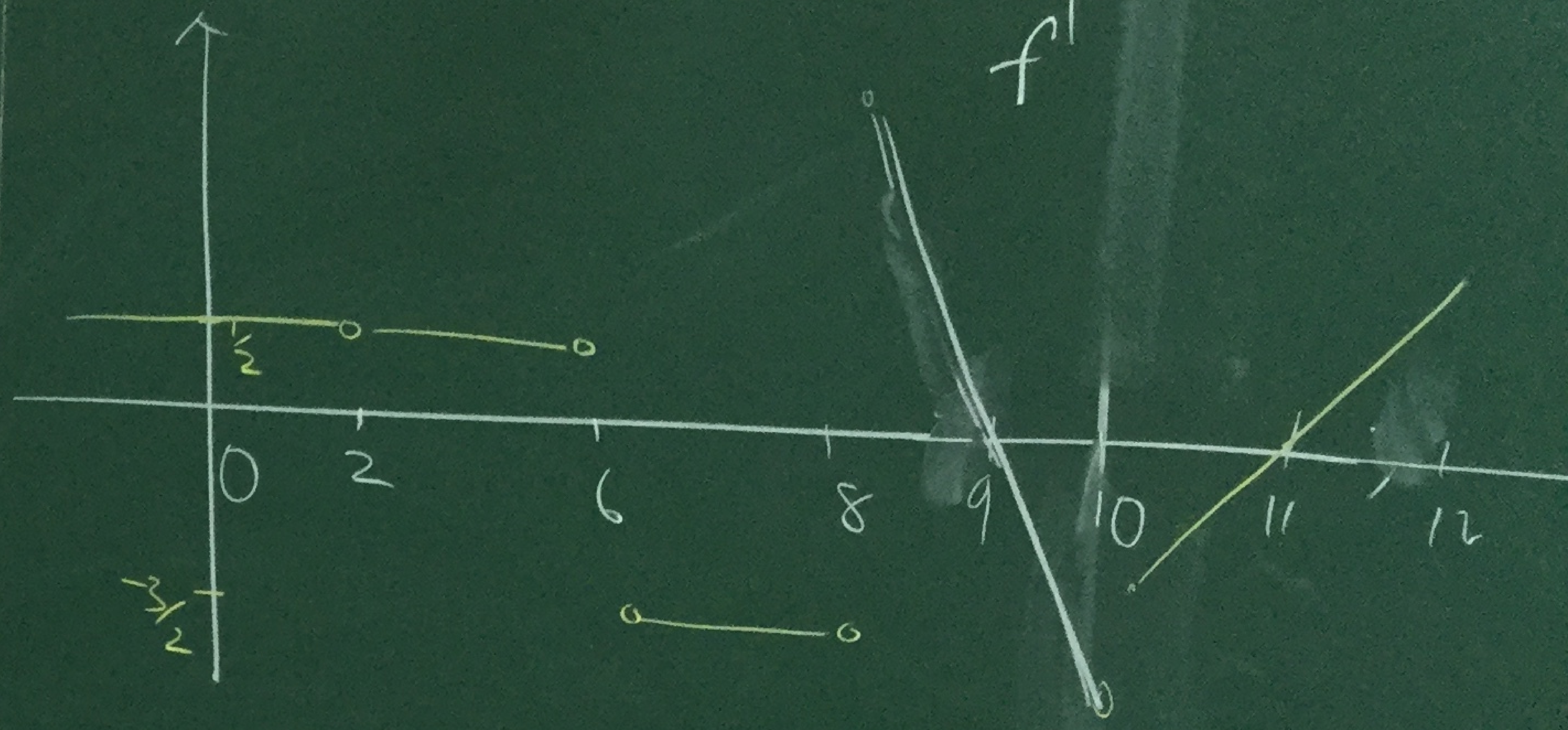
$x=2$, $x=6$, $x=8$, $x=10$,

f is not continuous at $x=2$

LHD of $f = \frac{1}{2}$ at $x=6$

RHD of $f = -\frac{3}{2}$ at $x=6$

from picture



3. $f : [0,4] \to \mathbb{R}$

$$f(x) = x^2 - 6x + 10$$

Fact: $f : [a,b] \to \mathbb{R}$, f is continuous

$\Rightarrow f$ attains the maximum & minimum

- Find Critical points

$$f'(x) = 2x - 6 = 2(x-3)$$

$$f'(x) = 0 \Rightarrow 2x - 6 = 0 \Rightarrow \boxed{x = 3}$$

Signchart

$0\ - - - - -\ 3\ + + + + + +\ 4$

End points 0 and 4

Maximum & Minimum

Critical point: $f(3) = 9 - 18 + 10 = 1$

End point: $f(0) = 10$

$f(4) = 2$

As $f \downarrow$ decreasing in $(0,3)$

$f \uparrow$ increasing in $(3,4)$

5

Minimum at $x=3$

maximum at $x=0$

$\square$

5 (a) $f(x) = x^4 - 4x^2 = x^2(x^2-4)$

$= x^2(x-2)(x+2)$

Zeros of f, $x=0, x=2, x=-2$

(b) CRITICAL POINTS

$f'(x) = 4x^3 - 8x$

$= 4x(x^2-2)$

$= 4x(x-\sqrt{2})(x+\sqrt{2})$

CP $x=0$, $x=\sqrt{2}$, $x=-\sqrt{2}$

Signchart $- - - - 0 + + + + + 0 - - - - 0 + +$

$-\sqrt{2}$ 0 $\sqrt{2}$

$$f''(x) = 12x^2 - 8$$
$$= 12\left(x - \sqrt{\tfrac{8}{12}}\right)\left(x + \sqrt{\tfrac{8}{12}}\right)$$
$$= 12\left(x - \sqrt{\tfrac{2}{3}}\right)\left(x + \sqrt{\tfrac{2}{3}}\right)$$

Inflection points $\quad 12\left(x - \sqrt{\tfrac{2}{3}}\right)\left(x + \sqrt{\tfrac{2}{3}}\right) = 0$

$$\Rightarrow x = \sqrt{\tfrac{2}{3}}, \quad x = -\sqrt{2/3}$$

Sign chart $\quad$ + + + + + 0 − − − − − − − 0 + + + + + + +

$\qquad\qquad\qquad -\sqrt{2/3} \qquad\qquad \sqrt{2/3}$

$\Rightarrow$ f is decreasing in $(-\infty, -\sqrt{2})$

f is increasing in $(-\sqrt{2}, 0)$

f is decreasing in $(0, \sqrt{2})$

f is increasing in $(\sqrt{2}, \infty)$

& $-\sqrt{2/3}$, $\sqrt{2/3}$ are inflection points <u>only</u>

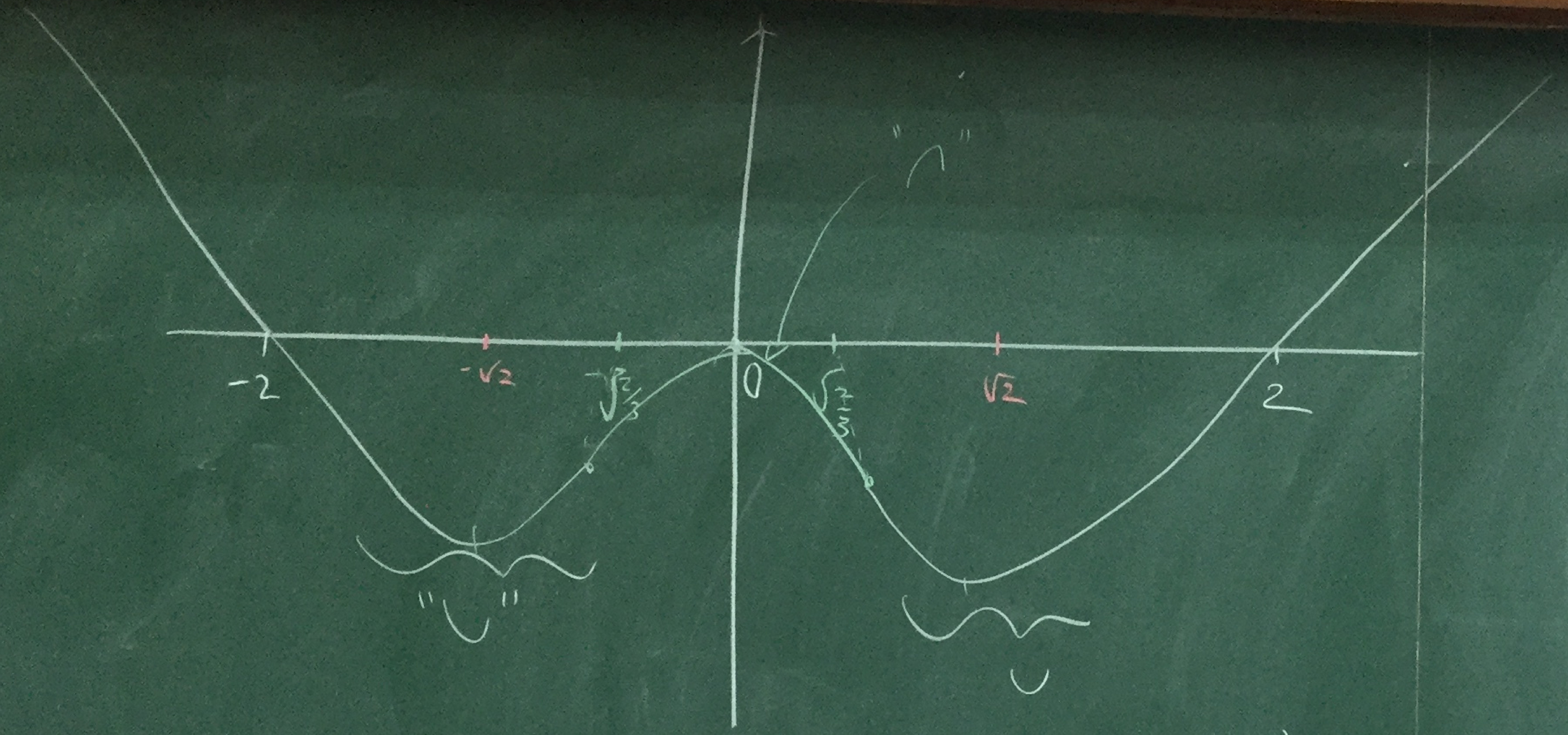
$\Rightarrow$ $-\sqrt{2}$ is local minima

0 is local maxima

$\sqrt{2}$ is local minima

Behaviou

points



<u>Behavior of y</u>: at ∞ $\quad \lim_{x\to\infty} x^4-4x^2 = \lim_{x\to\infty} x^2(x-2)(x+2) = \infty$

at $-\infty$ $\quad \lim_{x\to-\infty} x^4-4x^2 = \lim_{x\to-\infty} x^2(x-2)(x+2) = \infty$