

**Question 1:** Let

$$W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} : x_1 + x_2 + x_3 + x_4 = 0 \right\} \subset \mathbb{R}^4$$

- .
- (a) Show that  $W$  is a subspace of  $\mathbb{R}^n$ .
  - (b) Find a basis and the dimension of  $W$ .

**Question 2:** Find a LU decomposition for the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}.$$

Use the above decomposition to find a solution of the system of equations  $AX = B$  where

$$B = \begin{bmatrix} 1 \\ 6 \\ 8 \end{bmatrix}.$$

**Question 3:** Find the nullspace and the range of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}.$$

**Question 4:** Find eigenvalues and corresponding eigenvectors of the matrix

$$\begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}.$$

Describe the eigenspaces corresponding to each eigenvalue.