**May 9th, 2018** **Name (Please Print)** ________________________

**SWMS-Homework in Calculus** **Extra**

**Practice Problems:**

1. Find the domain of the funciton $f(x) = \dfrac{\sqrt{x^2-4}}{5-\sqrt{36-x^2}}$.

2. Let $f(x) = \begin{cases} 1 & \text{if } x < 0 \\ x & \text{if } 0 < x < 1 \\ 2-x & \text{if } 1 < x < 3 \\ x-4 & \text{if } x > 3 \end{cases}$.

   (a) Is it possible to define $f$ at $x = 0$ in such a way that $f$ becomes continuous at $x = 0$?

   (b) Is it possible to define $f$ at $x = 1$ in such a way that $f$ becomes continuous at $x = 1$?

   (c) Is it possible to define $f$ at $x = 3$ in such a way that $f$ becomes continuous at $x = 3$?

3. Give an example of a function defined on $[0, 1]$ which has no maximum and no minimum on the interval.

4. We say $\lim_{x\to 0} f(x) = 0$ if

   *For every $\epsilon > 0$ there exists $\delta > 0$ such that $| f(x) | < \epsilon$ whenever $| x | < \delta$.*

   Consider the following statements:

   (a) For every $\epsilon > 0$ there exists $\delta > 0$ such that for all $x \in \mathbb{R}$, $| x | < \delta$ implies $| f(x) | < \epsilon$.

   (b) For every $\delta > 0$ there exists $\epsilon > 0$ such that for all $x \in \mathbb{R}$, $| x | < \delta$ implies $| f(x) | < \epsilon$.

   (c) There exists $\delta > 0$ such that for all $\epsilon > 0$ and for all $x \in \mathbb{R}$, $| x | < \delta$ implies $| f(x) | < \epsilon$.

   (d) For every $\epsilon > 0$ and for all $x \in \mathbb{R}$, there exists $\delta > 0$ such that $| x | < \delta$ implies $| f(x) | < \epsilon$.

   Decide which of the above versions are equivalent to the definition of $\lim_{x\to 0} f(x) = 0$ and which are not. Give an example of $f$ that satisfies each of the above conditions.

5. Find $a$ so that the function $f$ is continuous at origin.

$$f(x) = \begin{cases} \frac{1-\cos 4t}{t^2} & \text{if } t < 0 \\ \frac{2(x+8)}{\sqrt{16-\sqrt{x}}} & \text{if } t > 0 \\ a & \text{if } t = 0. \end{cases}$$

6. In economics, The usefulness or *utility* of amounts $x$ and $y$ of two capital goods $G_1$ and $G_2$ is sometimes measured by a function $U(x, y)$. For example, $G_1$ and $G_2$ might be the two chemicals pharmaceutical company needs to have on hand and $U(x, y)$ the cost of manufacturing a product whose synthesis requires different amounts of the chemicals depending on the process used. The company wants to minimize $U$ when each unit of $G_1$ costs Rs 2 per kilogram, each unit of $G_2$ costs Rs 1 per kilogram, and the total amount allocated for the purchase of $G_1$ and $G_2$ together is Rs 30,

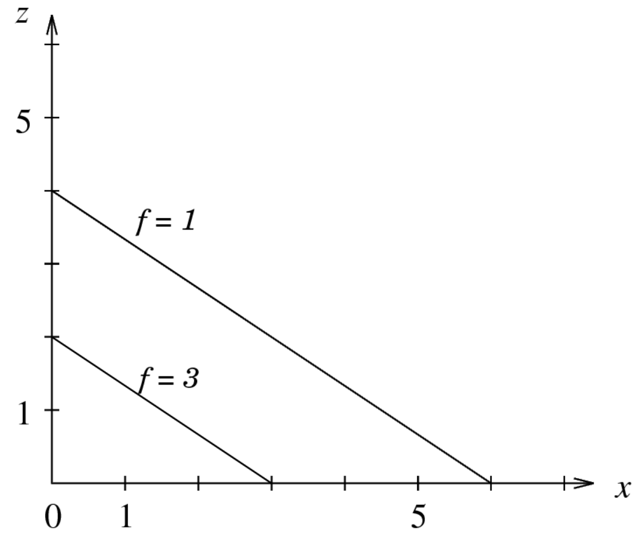
**May 9th, 2018** **Name (Please Print)** ____________________

**SWMS-Homework in Calculus** **Extra**

7. Given $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{0, 1, 4, 5\}$. Find the following sets.
   $A \setminus B$, $B \setminus A$, $A \times B$, $\mathcal{P}(B)$ and $B \times \mathbb{N}$

8. Solve the equation $| x - 2 |^2 + 3 | x - 2 | - 4 = 0$.

9. $f(x, y)$ is a linear function of two variables.




   (a) Find an expression for $f(x, y)$.
   (b) Draw the level curve of $f$ at level 0

10. Find the minimum value of $x_1 + x_2$, subject to the constraint $x_1 x_2 = 16$

11. Find the maximum value of $x_1 x_2$ subject to the constraint $x_1 + x_2 = 16$.

12. Let $g(x, y) = xy + \frac{8}{x} + \frac{1}{y}$
    (a) Find all critical points of $f(x, y)$ in the plane.
    (b) Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum or a saddle point.

13. Find the gradient of the function $r(x, y) = \sqrt{(x^2 + y^2)}$ and $\rho(x, y, z) = \sqrt{(x^2 + y^2 + z^2)}$ .
    Find the gradient of $\frac{1}{\rho}$.