1. Prove that if A and B are any two square matrices, and AB is nonsingular, then both A and B are nonsingular.

2. Show that $rank(A) = rank(P^{-1}AP)$ for any square matrix A and any invertible matrix P.

3. Let
$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}.$$
(a) Show that the eigenvalues of A are 1 and 2.
(2) Determine a basis of $\mathbb{R}^3$ of eigenvectors. Then give an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.

4. Consider
$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$
as a matrix over $\mathbb{C}$.
(a) Find the characteristic polynomial of A. Find the eigenvalues of A.
(b) Find an invertible complex matrix P such that $P^{-1}AP$ is diagonal.

5. Find all 2×2 real matrices such that $A^2 = I$, and describe geometrically the way they operate by left multiplication on $\mathbb{R}^2$.