**Question 1:** Prove that elementary matrices are invertible. Show that their inverses are also elementary matrices.

**Question 2:** Consider the three planes

$$x + 2y + 5z = 7 \qquad 2x - y = -1 \qquad 2x + y + 4z = k$$

(a) For which values of the parameter $k$ do these three planes have at least one point in common?
(b) Determine the common points.

**Question 3:** Show that if $A'$ is obtained from $A$ by a sequence of elementary row operations, and $B'$ is obtained from $B$ by applying the **same** sequence of row operations, then the system $A'X = B'$ has the same solutions as $AX = B$.

**Question 4:** Find bases for the null space and row space of the matrix

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}.$$

**Practice Problems**

**Question 5:** Define what is meant by the *dimension* of a subspace. Let $W$ be a subspace of $\mathbb{R}^n$. Give three statements about $W$ that are equivalent to the statement that $\dim(W) = p$.

**Question 6:** Let $A_1, A_2, A_3$ be the columns of a $4 \times 3$ matrix $A$ and let $b = [\,b_1\ b_2\ b_3\ b_4]^t$. Let the row reduced echelon form of the augmented matrix $(A|b)$ be

$$(A'|b') = \begin{pmatrix} 1 & 0 & 0 & -0b_2 + b_1 \\ 0 & 1 & 0 & b_2 - 2b_1 \\ 0 & 0 & 1 & -3b_3 + b_1 + 7b_2 \\ 0 & 0 & 0 & 5b_4 - 19b_2 + 2b_3 - b_1 \end{pmatrix}.$$

What condition does $b$ need to satisfy such that $b$ is in the $\text{Span}\{A_1, A_2, A_3\}$? Decide whether $v = [\,5\ \ 0\ 0\ 1]^t$ and $w = [\,0\ \ -1\ 0\ 1]^t$ are in $\text{Span}\{A_1, A_2, A_3\}$. Justify.

**Question 7:** Determine whether the following three vectors are linearly dependent or linearly independent:

$$u = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}, \quad v = \begin{bmatrix} 17 \\ -5 \\ 5 \end{bmatrix}, \quad \text{and} \quad w = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}.$$

If these vectors are linearly dependent, describe a nontrivial linear combination that yields the zero vector.

**Question 8:** Find a LU decomposition for the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}.$$

**Question 9:** Show that if $A$ is an invertible $n \times n$ matrix, then $A : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ takes a linearly independent set of vectors to another linearly independent set of vectors.

**Question 10:** Let $A_{m\times n}$ and $B_{n\times p}$ be two matrices. Show that the $\text{rank}(AB) \leq \text{rank}(A)$.