Question 1: Let $\{(x_i, y_i) : 1 \leq i \leq n\}$ be a set of points on the plane. Let $a, b \in \mathbb{R}$

(a) Let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix} \text{ and } X = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \cdots & \\ x_n & 1 \end{bmatrix}.$$

Show

$$\sum_{i=1}^{n} (y_i - (ax_i + b))^2 = \left(Y - X \begin{bmatrix} a \\ b \end{bmatrix}\right)^T \left(Y - X \begin{bmatrix} a \\ b \end{bmatrix}\right)$$

(b) Identify a, b that minimizes $\sum_{i=1}^{n} (y_i - (ax_i + b))^2$ as a solution to

$$X^T X \begin{bmatrix} a \\ b \end{bmatrix} = X^T Y.$$

(c) Suppose we are given $\left\{ \left(z_i, \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right) : 1 \leq i \leq n \right\}$. What linear system does a, b, c have to solve: so as to minimize $\sum_{i=1}^{n} (z_i - (ax_i + by_i + c))^2$?

Question 2: Let $f(x, y) = \frac{x^2}{2} + 2y^2$

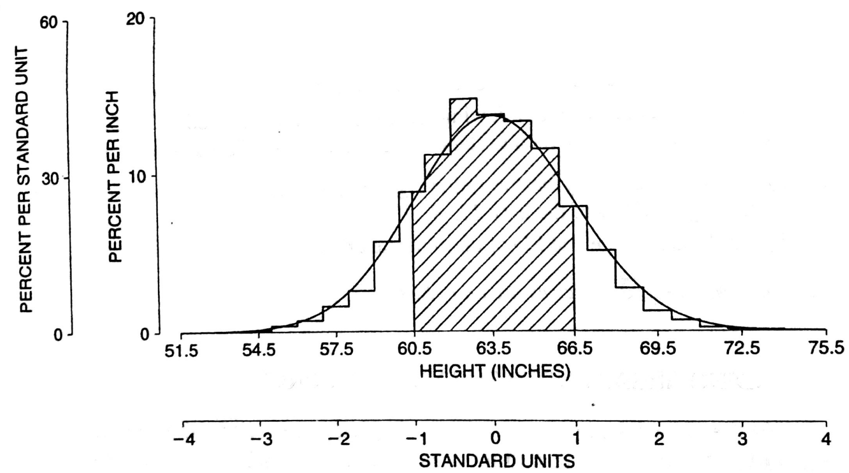
(a) Can you guess a minima for f ?

1. Draw the level curves of f at levels $1, 10, 100$.

(b) Let $x^{(0)} = (4, 1)$. Find a suitable t_k for $k \geq 1$. Calculate $x^{(k)}$ using Gradient Descent algorithm.

(c) Does $x^{(k)}$ converge and if so where ?

Question 3: The following is a histogram for heights of women compared to a normal curve. The average height of women is 63.5 inches and standard deviation is 3 inches. Using the normal



tables approximate

(a) the area under the histogram between 60.5 inches and 66.5 inches.

(b) the area under the histogram above 59 inches.