Question 1: Consider the following data set:

$$
\begin{gathered}
84,49,61,40,83,67,45,66,70,69,80,58,68,60,67,72,73,70,57,63,70,78,52,67,53,67, \\
75,61,70,81,76,79,75,76,58,31 .
\end{gathered}
$$

1. Find the mean, median and standard deviation of the data.
2. Fill in the following frequency table for the above data set:

| Classes | Frequency |
| :---: | :---: |
| $0-35$ |  |
| $36-45$ |  |
| $46-55$ |  |
| $56-65$ |  |
| $66-75$ |  |
| $76-85$ |  |

3. Draw the histogram of the above data on the graph below:

Question 2: (a) Three histograms are given below. Fill in the blank for each of the histogram: The average is around $\qquad$ Options : $-1,0,4,5,-3$

(b) Three histograms are given below, all with mean 0 . Which of these has maximum and minium variance?


Question 3: Let $\left\{\left(x_{i}, y_{i}\right): 1 \leq i \leq n\right\}$ be a set of points on the plane. Let $a, b \in \mathbb{R}, \bar{x}=\frac{\sum_{i=1}^{n} x_{i}}{n}$ and $\bar{y}=\frac{\sum_{i=1}^{n} y_{i}}{n}$

1. Show

$$
\sum_{i=1}^{n}\left(a x_{i}+b-y_{i}\right)^{2}=\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}+a^{2} \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}-2 a \sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)+n(\bar{y}-a \bar{x}+b)^{2} .
$$

2. Identify $b$ that minimizes $\sum_{i=1}^{n}\left(a x_{i}+b-y_{i}\right)^{2}$
3. Proceed to find $a$ that minimizes $\sum_{i=1}^{n}\left(a x_{i}+b-y_{i}\right)^{2}$.
$\qquad$

Question 4: Consider the following data:

| x | y |
| :---: | :---: |
| 2 | 6 |
| 4 | 8.5 |
| 1 | 2.5 |
| 7 | 15 |
| 5 | 11 |

(a) Make a scatter plot of $(x, y)$.

1. Suppose the line that gives a best fit is:

$$
y=a x+b
$$

then the predicted value of $y$ for $x_{i}$ is given by $\hat{y}_{i}=a x_{i}+b$, and the prediction errors are defined by $d_{i}=y_{i}-\hat{y}_{i}$. Using answer found in Question 3, find the values of $a$ and $b$ which minimizes the error sum of squares which is $\sum_{i=1}^{n} d_{i}^{2}$.
(b) For the values of $a$ and $b$ obtained, draw the line $y=a x+b$ on the graph above and fill in the following table:

| x | y | $\hat{y}$ | $d^{2}$ |
| :---: | :---: | :---: | :---: |
| 2 | 6 |  |  |
| 4 | 8.5 |  |  |
| 1 | 2.5 |  |  |
| 7 | 15 |  |  |
| 5 | 11 |  |  |

(c) Estimate the value of $y$ when $x=8,3$.

