Question 1: (a) Let us define the following terms:-

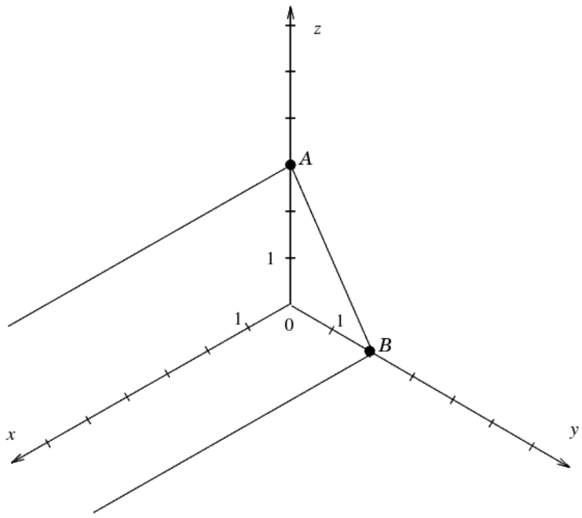
- S is bounded if there exists M such that $\mid x \mid \leq M$ for all $x \in S$.
- f is increasing (or strictly increasing) if $f(x) < f(y)$ whenever $x < y$.

(i) Give an Example of S and express the statement: S is not bounded

(ii) Give an Example of f and express the statement: f is not increasing (i.e., the negation of the increasing property).

(b) Find the domain of $g(x) = \sqrt{f(x)}$, where $f(x) = \dfrac{\sqrt{3-x}}{2+x}$.

Question 2: The plane γ is shown below on the graph.



It is parallel to the $x - axis$ and passes through the points A and B.

1. Find the normal vector $\overrightarrow{n}$ to γ and write an equation for γ.
2. Find the distance from the point $(3, -3, 1)$ to γ.

Question 3 Let $f(x, y) = x^2 - 2x^3 + 3y^2 + 6xy$

(a) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial^2 x}$, $\frac{\partial^2 f}{\partial^2 y}$ and $\frac{\partial^2 f}{\partial x \partial y}$.

(b) Find all critical points of $f(x, y)$ in the plane.

(c) Use the second derivative test to determine (if possible) whether each critical point is a local maximum, a local minimum or a saddle point.

Question 4 Meet *Squirmy*, she is a ladybug. One day Squirmy decides to explore the fabulous ICTS campus.

(a) At first, Squirmy starts walking along the curve C in the $x-y$ plane, given by the equation:

$$\frac{x^2}{400} + \frac{y^2}{100} = 1.$$

On reaching $(12, 8)$, she is supposed to start walking in a direction perpendicular to C. What direction should she move in ?

(b) Next Squirmy jumps into the Kitchen via the windo. She lands on a hot plate where dosai is being made, ***Ouch!!***. She is at a position $(1, 1)$ and suppose the temperature is given by

$$T(x, y) = 1000\mathrm{e}^{-x^2-y^4}.$$

(i) What direction should she move in order to decrease the temperature the fastest ?

(ii) Instead, she moves in the direction $(2, 1)$. Use a linear approximation to estimate the temperature of the plate when Squirmy gets to $(1.02, 1.01)$.