Question 1: Find whether the following set of vectors $S = \{v_1, v_2, v_3\}$ is linearly independent or not, where

$$v_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \qquad v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}.$$

Question 2: Find the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & -3 \\ -3 & 6 & 5 \end{bmatrix}.$$

Question 3: Let X be an eigenvector of an $n \times n$ matrix A, with nonzero eigenvalue λ. Prove that if A is invertible, then X is also an eigenvector of A^{-1}, with eigenvalue λ^{-1}.

Question 4: Show that 0 is an eigenvalue of a matrix A if and only if A is singular, ie. $\det(A) = 0$.