Shape of the Earth: Method of least Squares

Credits: Talk (borrowed from)- Probal Chaudhuri's IMS public lecture

May 13, 2018

What is the Shape of the Earth ?

What is the Shape of the Earth ?

- French astronomer Jean Reicher in South America in 1672
- Reichers pendulum clock was losing 2 minutes 28 seconds every day in French Guyana.

- Isaac Newton and his Principia in 1687
- Newton claimed that the Earths rotation around its axis made it an Oblate Sphere. Rotation caused flattening at the Poles and bulging at the Equator.

- Was not unanimously regarded as correct

Side View of oblate earth : *Berry, 1898, p 277*

. . . lengthening of degrees arc toward the pole . . .

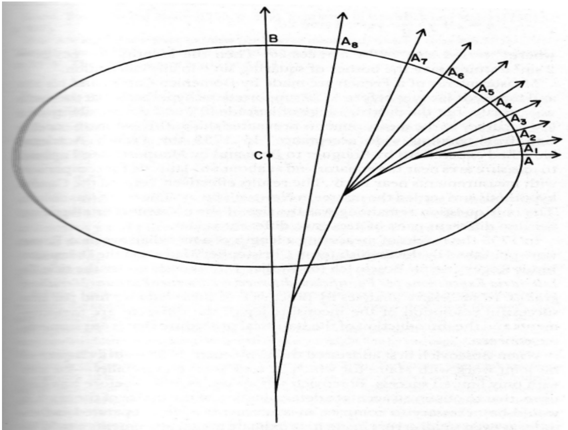


Figure 1.3. A side view of an exaggeratedly oblate earth, illustrating the lengthening of degrees of arc toward the pole. The meridian quadrant AB is broken into nine segments, each of 10° latitude. (Based upon Berry, 1898, p. 277.)

. . . meridian quadrant AB is broken into 9 segments of 10°

Arc Length and the Latitude

- Let Y be the length of a 1° meridian arc centered at latitude θ.
- Then, assuming the Earth to be an ellipsoid,

$$Y \approx m\sin^2(\theta) + c = mX + c,$$

  where $X = \sin^2(\theta)$.
- Ellipticity of the Earth can be measured by $\frac{m}{3c}$.
- Newton estimated the ellipticity of the Earth to be $\frac{1}{230}$.

Newton and the French Scientists

- French arc measurements around 1720 over a range of 9° latitude. The project was led by Dominico Cassini and Jacques Cassini of the Royal Observatory in Paris.
- Their measurements and analysis contradicted Newtons model for the Earth.
- French Geodesic Mission during 1735 – 1736. Expeditions to Quito in Ecuador and Lapland in Finland.

Bosocovitch Data on Meridian Arc

Table 1.4. Boscovich's data on meridian arcs.

Location	Latitude (θ)	Arc length (toises)	Boscovich's $\sin^2\theta \times 10^4$
(1) Quito	0°0′	56,751	0
(2) Cape of Good Hope	33°18′	57,037	2,987
(3) Rome	42°59′	56,979	4,648
(4) Paris	49°23′	57,074	5,762
(5) Lapland	66°19′	57,422	8,386

Source: Boscovich and Maire (1755, p. 500). Reprinted in Boscovich and Maire (1770, p. 482).

Note: Arc lengths are given as toises per degree measured, where 1 toise ≅ 6.39 feet. The value for $\sin^2\theta \times 10^4$ for the Cape of Good Hope is erroneous and is evidently based on 33°8′. The correct figure would be 3,014.

Bosocovich: Least Absolute Mean Deviation(1755)

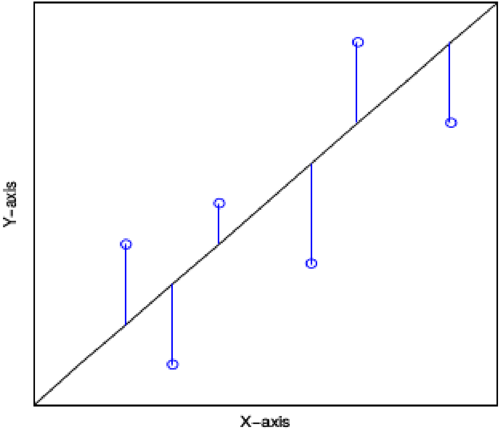
We have data points

$$(X_1, Y_1), (X_2, Y_2), \ldots, (X_5, Y_5).$$

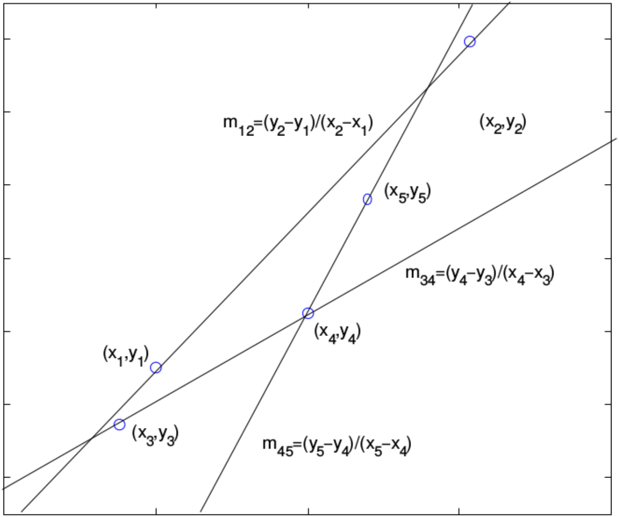
where Y reprsents arc length, and X is $\sin^2$ (latitude).
To find the line : $Y = mX + c$ that fits the data best.

$$\min\{m, c : \sum_{i=1}^{5} |Y_i - mX_i - c|\}.$$

Least Absolute Deviation Line



Bosocovich : Pairwise Solution



Bosocovich : Pairwise Solution

Table 1.5. Boscovich's pairwise solutions, based on the data of Table 1.4, for the polar excess y (the amount by which a degree at the pole exceeds a degree at the equator) and the ellipticity (found from the formula $1/\text{ellipticity} = 3z/y$, where z is the length of a degree at the equator as found from the pair of equations).

Pair	Polar excess (y, in toises)	Ellipticity	Pair	Polar excess (y, in toises)	Ellipticity
1, 5	800	1/213	2, 4	133	1/128
2, 5	713	1/239	3, 4	853	1/200
3, 5	1,185	1/144	1, 3	491	1/347
4, 5	1,327	1/128	2, 3	−350	−1/486
1, 4	542	1/314	1, 2	957	1/78

Source: Boscovich and Maire (1755, p. 501). Reprinted in Boscovich and Maire (1770, p. 483).

Note: The ellipticities for pairs (2, 4) and (1, 2) were evidently misprinted in the original; they should be 1/1282 and 1/178. The figures for the pair (1, 4) are erroneous; they should be 560 and 1/304.

Laplace (1799)

- **Question:** How to minimize for m in $\sum_{i=1}^{n} \mid Y_i - m \mid$?

- **Question :** How to minimize $\sum_{i=1}^{n} \mid Y_i - mX_i \mid$ w.r.t. m ?

- **Question :** How to minimize $\sum_{i=1}^{n} \mid Y_i - mX_i - c \mid$ w.r.t. m and c?

Laplace Solution (1799)

- **Question:** How to minimize for m in $\sum_{i=1}^{n} \mid Y_i - m \mid$?
  - **Answer :** The minimizing m will be the median.
- **Question :** How to minimize $\sum_{i=1}^{n} \mid Y_i - mX_i \mid$ w.r.t. m ?
  - **Answer :** Observe:

$$\sum_{i=1}^{n} \mid Y_i - mX_i \mid = \sum_{i=1}^{n} \mid \frac{Y_i}{X_i} - m \mid\mid X_i \mid .$$

    Let $Z_i = \frac{Y_i}{X_i}$, then m will median of Z_i with the weight distribution $\mid X_i \mid$

- **Question :** How to minimize $\sum_{i=1}^{n} \mid Y_i - mX_i - c \mid$ w.r.t. m and c? (Provided partial answer)

What is one Meter ?

Metric System : the origin of one meter

- In 1792, after the French Revolution, France and many other European counties decided switch to a new system of measurement - the Metric System.

- One Meter was defined to be $1/10,000,000$ of a Meridian Quadrant, which is the distance from the Equator to the North Pole.

Adrien-Marie Legendre (1752-1833)

- In 1792, Adrien Marie Legendre, was associated with the French Commission charged with the measurement of a Meridian Quadrant.

Legendre invented and published the Principle of Least Squares in 1805.

Legendre: How to minimize

$$\sum_{i=1}^{n} (Y_i - mX_i - c)^2$$

w.r.t. m and c?

French Meridian Arc Measurements

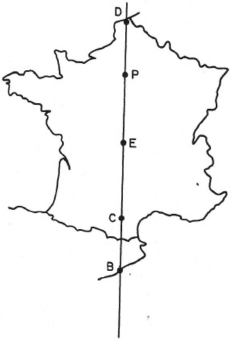


Fig. 1. *The French meridian arc, through Dunkirk* (D), *the Pantheon* (P) *in Paris, Evaux* (E), *Carcassone* (C), *and Barcelona* (B).

Dunkirk to Paris to Evaux to Carcassone to Barcelona

French Meridian Arc Measurements

TABLE 1.

French arc measurements, from Allgemeine Geographische Ephemeriden, 4, 1799, *page* xxxv. *The number* 76545.74 *is a misprint; the correct number is* 76145.74. *The table gives the length of four consecutive segments of the meridian arc through Paris, both in modules* S *(one module* ≅ 12.78 *feet) and degrees* d *of latitude (determined by astronomical observation). The latitude of the midpoint* L *of each arc segment is also given.*

	Modules S	Degrees d	Midpoint L
Dunkirk to Pantheon	62472.59	2.18910	49° 56′ 30″
Pantheon to Evaux	76545.74	2.66868	47° 30′ 46″
Evaux to Carcassone	84424.55	2.96336	44° 41′ 48″
Carcassone to Barcelona	52749.48	1.85266	42° 17′ 20″
Totals	275792.36	9.67380	

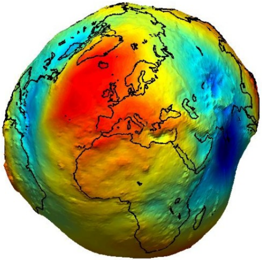
Carl Friedrich Gauss (1777-1855)

- In January-February 1801, Italian astronomer Joseph Piazzi observed and recorded data on the dwarf planet Ceres for 41 days before it vanished in the glare of the Sun.
- Astronomers wanted to predict the position and the time of re-appearance of the Ceres. Many scientists including Laplace thought that it is an impossible problem due to inadequate data.
- Gauss made a very accurate prediction of the time and the position of re-appearance. The Ceres was observed again in November-December 1801.

Carl Friedrich Gauss (1777-1855)

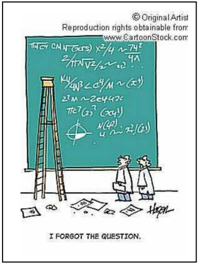
- Gauss finished writing his work related to the Method of Least Squares in 1807, and the book was published in 1809.
- Gauss claimed Least Squares as his method and said that he was using it since 1795.
- This is one of the most famous priority disputes in the history of Statistical Science.

The Actual Shape of the Earth



ESA's GOCE mission has delivered the most accurate model of the 'geoid' ever produced, which will be used to further our understanding of how Earth works.

Asking the right Question



Important:

Ask the right question

Understanding is achieved.

Successes in many areas